MATHEMATICAL STUDIES

ANSWERS

Editor: Fabio Cirrito

4th Edition

FOR USE WITH THE I.B. DIPLOMA PROGRAMME

ANSWERS

Exercise 4.2

1 **a** 4 **b** 5 **c** 2 **d** 4 **e** 3 **f** 3 **g** 5 **h** 3 **i** 6 **j** 4 **k** 6 **l** 5
2 **a** 2.5 **b** 24700 **c** 0.35 **d** 45630 **e** 0.45 **f** 4 **g** 57000 **h** 0.0454 **i** 0.0045 **j** 346000 **k** 0.045 **l** 90000
3 8.2 cm
4 11 cm
5 44 m
6 1040 mm
7 30 terms
8 3 S.F.
9 19

Exercise 4.3

1 **a** 2.2×10^{1} **b** 1.2×10^{-12} **c** 4.8×10^{9}
2 $r = 4.72 \times 10^{9}$m $C = 2.97 \times 10^{10}$m , Vol = 4.40×10^{29}m^3
3 7.48×10^{-6}m^3
4 280 km
5 4.494×10^{15} kgm^{2}s^{-2}
6 3.4×10^{9}
7 74 generations, 1.9×10^{22} relations.
8 1.25×10^{8} (to 3 S.F.) or 1.3×10^{8} (to 2 S.F.)
9 3.1×10^{-24} gms
10 4.3×10^{6}

Exercise 4.4

1 0.0033333 (etc., 0.01, 1%)
2 0.0144, 0.0053, 0.53%
3 The absolute errors are all 0.2 kg. The relative and percentage errors are: **a** 0.027 (2.7%) **b** 0.0102 (1.02%) **c** 0.00893 (0.893%) **d** 0.00352 (0.352%) **e** 0.00202 (0.202%)
4 Absolute errors are all 0.5 mm. **a** 0.1 (10%) **b** 0.05 (5%) **c** 0.00417 (0.417%) **d** 0.002 (0.2%)
5 **a** 0.018% **b** 0.0014% **c** 0.273% **d** 0.0030%
6 **a** 0.45% **b** 0.073% **c** 0.020% **d** 0.0022%
7 6.6×10^{-4} % which is plainly far more accurate than can be justified in the times quoted for this flight.

Exercise 4.5

1 **a** 0.209 [0.204, 0.215] **b** 3384 [1726, 64752] **c** –194.9 [–206.3, –184.85]
2 516577 mm^2 [514157, 519000] 0.43%
3 164553 cm^2 [150456, 179503] 9.1%
4 17961 [17936, 17986]
5 28.24 m [27.39, 29.11] 3.1% accuracy.
6 105.6 m [104.9, 106.2] 0.66%
7 $a \approx b$
8 0.25 kg
9 15.42 cm [14.90, 15.97] 3.6%
10 12096 [10646, 13672] 13%
11 2208 [2076, 2344] 6.2%
12 0.00069%

Exercise 4.6 Miscellaneous questions

1 **a** 0.380568 **b** 0.381 **c** 0.38 d 3.80568×10^{-1}
2 **a** 24548.616 **b** 24500 **c** 24548.62 **d** 24549 **e** 2.4548616×10^{4}
3 **a** **i** 75.0 **ii** 74.97 **iii** 75 **b** 7.5×10^{1}
4 **a** 790 **b** 790.02 **c** 790
5 $\frac{1}{7} = 0.14286$ to 5 S.F., absolute error = 2.86×10^{-6} % error = 0.0020
6 ≈ 0.20467227 or 0.205 to 3 S.F.
7 **a** 3 **b** 3 **c** 2
8 **a** 4.51×10^{21} **b** 6.01×10^{-17} **c** 2.12×10^{-1}
9 **a** 34000 **b** 0.000705
10 **a** 19.9 ± 0.8 **b** 45.44 or 41.54 to 49.5 **c** ≈ 1.429, 0.769 to 10
11 Series gives 2.96339 error (–0.1782) (5.7%)

4.7 Graded revision questions

LEVEL 1

1 **a, b, c, e**
2 **a** 3.4×10^{1} **b** 5.67×10^{4} **c** 6.00056×10^{5} **d** 4×10^{-1} **e** 4.38×10^{-3} **f** 4.0456×10^{0} **g** 2.0×10^{-3} **h** 7.5×10^{-1} **i** 5.4×10^{-6} **j** 1×10^{4} **k** 6×10^{-2} **l** 7.5×10^{-2}

LEVEL 2

1 **a** 7.92×10^{3} **b** 1.96×10^{2} **c** 3.15×10^{-2} **d** 4.83×10^{1} **e** 2.45×10^{-6}
2 **a** 3.3% **b** 11% **c** 6.9% **d** 11% **e** 11%
3 **a.** $\frac{13}{10}$ **b** $\frac{1}{4}$ **c** $\frac{1}{3}$ **d** $\frac{5}{9}$ **e** $\frac{5}{6}$ **f** $\frac{83}{99}$

LEVEL 3

1 **a** $v = \frac{uf}{u-f}$ **b** Smallest value = 0.46735765, Largest value = 0.54967293
c 8.12 %

2 **a** 1.8×10^{16} kg m^2 sec^{-2} **b** 0.0007515 J

3 **a** 76.4 cm^3 **b** 76.43 cm^3 **c** 76 cm^3.

LEVEL 4

1 $k = m(2+\sqrt{2}), m \in \mathbb{Q}$.

2 6.6 %

3 The percentage error is $\frac{150028}{14797665}(100) = 1.0138626$ or about 1%.

4.8 Topic test

1 **a** 1.45×10^{2} **b** 1.08×10^{11} **c** 1.59×10^{-6}

2 **a** 4010 **b** 0.00506 **c** 109000

3 $\frac{1}{3}\% \approx 0.33\%$

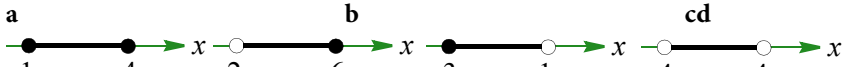
4 **a** 0.319024 **b** 0.319 **c** 0.32 **d** 3.19024×10^{-1}

5 $2.\dot{2}\dot{3} = \frac{221}{99}$. Therefore, $2.\dot{2}\dot{3} \in \mathbb{Q}$

6 **a** $9.41 \le A \le 10.05$ **b** % error = $3.275\% \approx 3.3\%$

Exercise 5.1

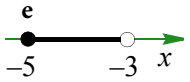
1 **a**



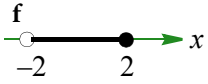
b

cd

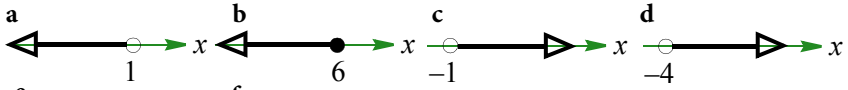
e



f



2 **a**

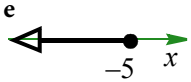


b

c

d

e



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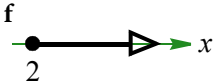
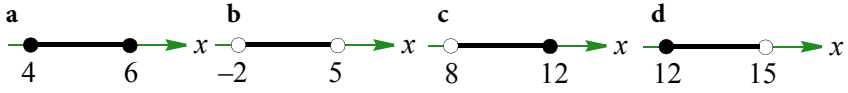


3 **a**]2, 8] **b** [4, ∞[**c** [0, 6] **d**]−∞, 10] **e**]−∞, 12[**f** [3, 5[**g**]−2, ∞[**h**]−3, 1] **i**]−∞, −7[

4 **a** $\{x|4 \le x \le 6\}$ **b** $\{x|-2 < x < 5\}$ **c** $\{x|8 < x \le 12\}$ **d** $\{x|12 \le x < 15\}$ **e** $\{x|5 \le x < \infty\}$ **f** $\{x|-\infty < x < 8\}$ **g** $\{x|2 < x < \infty\}$ **h** $\{x|-\infty < x \le 3\}$ **i** $\{x|-2 < x < \infty\}$

5 **a**

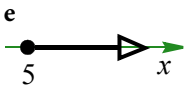


b

c

d

e



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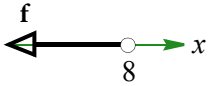
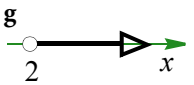
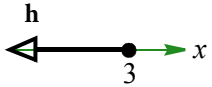


g



h



i

Exercise 5.2

1 **a** 5 **b** 5 **c** −3 **d** 21 **e** $\frac{28}{3}$ **f** $-\frac{19}{3}$ **g** 11 **h** 11 **i** 4

2 **a** 9 **b** 20 **c** 10 **d** $\frac{4}{3}$ **e** −9 **f** 15

3 **a** −1 **b** 2 **c** −5 **d** −1 **e** 1 **f** $\frac{11}{13}$

4 **a** $-\frac{13}{7}$ **b** $-\frac{7}{4}$ **c** 22 **d** $\frac{1}{31}$ **e** −19 **f** $\frac{16}{7}$ **g** $-\frac{9}{2}$ **h** 24.5

5 **a** $\frac{5}{6}$ **b** $-\frac{4}{3}$ **c** $-\frac{2}{3}$ **d** $\frac{11}{4}$ **e** $-\frac{1}{2}$ **f** $\frac{1}{4}$ **g** $\frac{7}{11}$ **h** $\frac{6}{5}$ **i** $\frac{4}{3}$ **j** $\frac{1}{2}$ **k** $\frac{5}{2}$ **l** $\frac{7}{6}$

6 **a** −0.5 **b** −1 **c** $\frac{13}{4}$ **d** $\frac{73}{11}$ **e** $\frac{9}{8}$ **f** −13 **g** $\frac{7}{3}$ **h** $-\frac{23}{2}$ **i** $\frac{1849}{150}$ **j** $-\frac{4}{3}$ **k** $\frac{3}{2}$ **l** $\frac{8}{5}$

7 **a** $\frac{a-2}{a}$ **b** $\frac{1+b}{b}$ **c** $-\frac{2}{a}$ **d** $\frac{5}{a}$ **e** $-\frac{2}{a}$ **f** $2a$ **g** $\frac{5}{4}(3-2a)$ **h** $1-a$ **i** $\frac{2}{3}\left(\frac{a-1}{a+1}\right)$

8 **a** $\frac{5}{a-b}$ **b** $\frac{a+b}{a-b}$ **c** 1 **d** −1 **e** $1-a$ **f** $2a-a^2$ **g** $\frac{ab}{a+b}$ **h** $\frac{a^2-b^2}{a-b} = a+b$ **i** 1

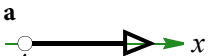
Exercise 5.3

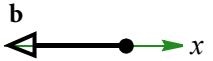
1 **a** $12 - y = 5$ **b** 7
2 **a** $4x + 2 = 30$ **b** 7
3 $2x - 5 = 19$, $x = 12$
4 12, 13
5 31, 32, 33
6 $x + 3x + 2x = 84$, 14, 28, 42
7 20
8 84
9 200
10 $3388.90
11 $\frac{4}{3}$
12 $13,739
13 **a** 7 mL **b** 23.33%
14 Tim−8 yrs, Claire−16 yrs, Thomas−10 yrs
15 3.75 km
16 22
17 5%
18 **a** 144 km **b** 40 kmh^{-1}
19 Ab −$12,500, Ba−$5,500
20 83.33%

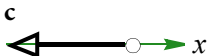
21 **a** $\frac{(2m+1)}{m(m+1)}d$ hr **b** $\frac{2m(m+1)}{2m+1}$ kmh^{-1}
22 32%
23 **a i** $10x+4$ **ii** $400+x$ **b** 364
24 60 kg

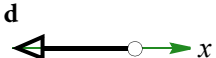
Exercise 5.4

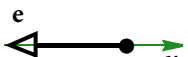
1 **a** $x>4$ **b** $x\leq 9$ **c** $x<4$ **d** $x<1$ **e** $x\leq 6$ **f** $x\leq -1$ **g** $x\geq 5$ **h** $x<5$ **i** $x\leq -3$
j $x>21$ **k** $x\geq \frac{28}{3}$ **l** $x>-\frac{19}{3}$ **m** $x\leq 11$ **n** $x<11$ **o** $x<-\frac{3}{2}$

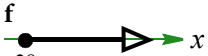
2 **a** 

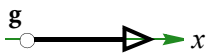
b 

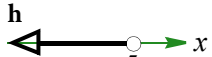
c 

d 

e 

f 

g 

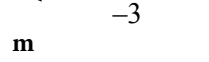
h 

i 

j 

k 

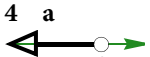
l 

m 

n 

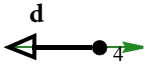
o 

3 **a** $x<9$ **b** $x\geq 20$ **c** $x>10$ **d** $x\leq \frac{4}{3}$ **e** $x>-9$ **f** $x\geq 15$

4 **a** 

b 

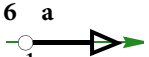
c 

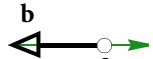
d 

e 

f 

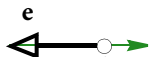
5 **a** $x>-1$ **b** $y<2$ **c** $x\leq -5$ **d** $x\geq -1$ **e** $x<1$ **f** $x<\frac{11}{13}$

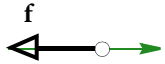
6 **a** 

b 

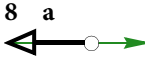
c 

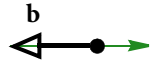
d 

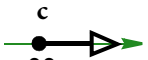
e 

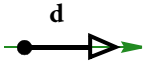
f 

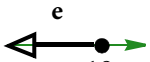
7 **a** $x<-\frac{13}{7}$ **b** $x\leq -\frac{7}{4}$ **c** $x\geq 22$ **d** $y\geq \frac{1}{31}$ **e** $x\leq -19$ **f** $w>\frac{16}{7}$

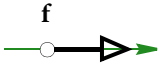
8 **a** 

b 

c 

d 

e 

f 

9 **a** $x<-27$ i.e. $]-\infty,-27[$ **b** $x\geq -12$ i.e. $[-12,\infty[$ **c** $x<\frac{180}{29}$ i.e. $]-\infty,\frac{180}{29}[$
d $x\geq -\frac{1}{9}$ i.e. $\left[-\frac{1}{9},\infty\right[$

10 $-5,-4,-3,-2,-1$

Exercise 5.5

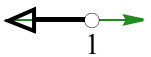
1 $x\geq 15$
2 $3\leq x<20$
3 $38<x<50$
4 40.5 kg
5 **a** $x>5$ **b** $x>4$
6 >\$2000
7 >25 kmh^{-1}
8 > 93 (or ≥ 94)

Exercise 5.6 Miscellaneous questions

1 $x=\frac{5}{2}$

2 **a** $x=-\frac{15}{4}$ **b** $x<\frac{5}{4}$

3 **a** $x\leq -\frac{9}{2}$ 

b $x<1$ 

4 **a** $\frac{27}{2}$ **b** 1

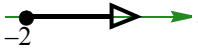
5 **a** $x>-5$

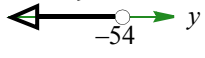
6 **a** $-\frac{5}{2}$ **b** $\frac{19}{5}$

7 23
8 26 five-cent coins.

9 **a** $a=7$ **b** $y=-2.4$ **c** $x=\frac{5}{3}$

10 **a** $x=6$ **b** $x<-\frac{13}{5}$

11 **a** $-2\leq x$ 

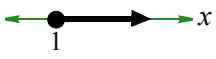
b $-54>y$ 

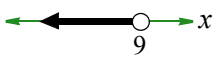
12 **a** $x=b+1$ **b** $x=ab$
13 3 litres

5.7 Graded revision questions

LEVEL 1

1 **a i** $\{x|4\leq x\leq 9\}$ $[4,9]$ **b i** $\{x|x>6\}$ **ii** $]6,\infty[$ **c i** $\{x|x<-3\}$ **ii** $]-\infty,-3[$
2 **a** **b** **c**





LEVEL 2

1 **a** $x = 4$ **b** $s = 3$ **c** $x = 1$ **d** $t = 4$ **e** $x = 4.5$ **f** $y = -\frac{10}{3}$

2 **a** $x > -1$ **b** $x \geq -4$ **c** $y < -6$ **d** $x > 2$ **e** $x < 7$ **f** $x > 0$.

LEVEL 3

1 **a** $x = 4$ **b** $x = 24.5$ **c** $s > 0.8$ **d** $y > \frac{17}{7}$ **e** $x = \frac{3}{2}$ **f** $a \geq -\frac{5}{11}$

2 Joseph is 22.

LEVEL 4

1 **a** $x = b - 1$ **b** If $ab > 0 \Rightarrow x > \frac{ab(a+b)}{a^2+b^2}$, if $ab < 0 \Rightarrow x < \frac{ab(a+b)}{a^2+b^2}$.

2 400 km. We have assumed that they travel in the same direction.

5.8 Topic test

1


2 **a** 2 **b** 5 **c** 4

3 **a** $x < 5$ **b**

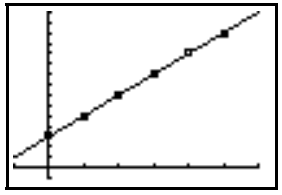

4 **a** −13 **b** $x \leq \frac{1}{23}$

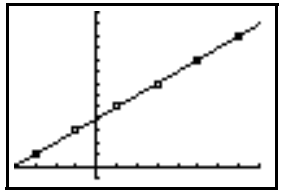
5 **a** $10 - x$ **b** $x = \frac{10}{3}$

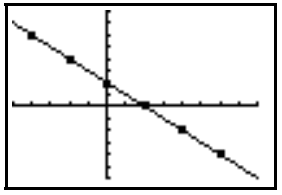
6 **b** $y = \frac{100}{31}$ **c** $x = 6200$

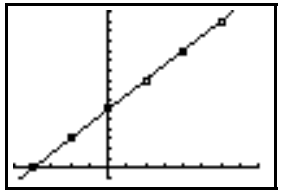
7 $w = 83\frac{1}{3}$

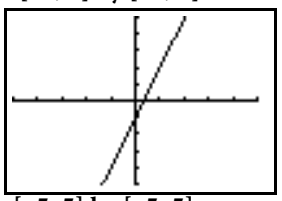
Exercise 6.1

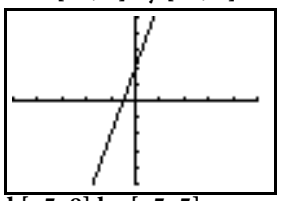
1 **a**


b


c


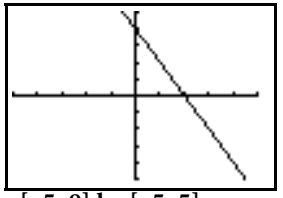
d


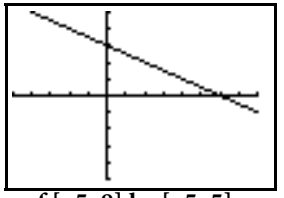
2 **a** [−5, 5] by [−5, 5]


b [−5, 5] by [−5, 5]


c [−5, 5] by [−5, 5]

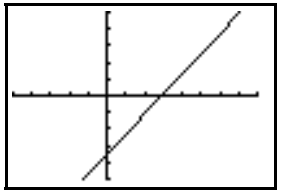
d [−5, 8] by [−5, 5]

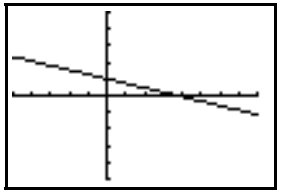
3




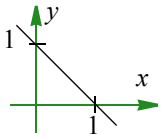
e [−5, 8] by [−5, 5]

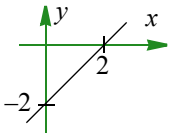
f [−5, 8] by [−5, 5]

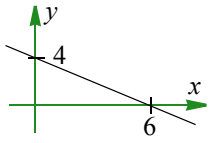
4


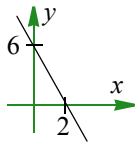


5 **a** $m = -1$, $c = 1$ **b** $m = 1$, $c = -2$ **c** $m = -\frac{2}{3}$, $c = 4$ **d** $m = -3$, $c = 6$

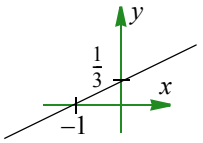


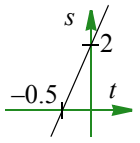




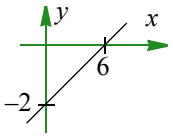


e $m = \frac{1}{3}$, $c = \frac{1}{3}$ **f** $m = 4$, $c = 2$

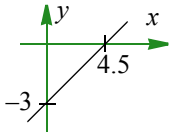




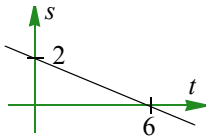
6 **a**



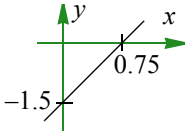
b



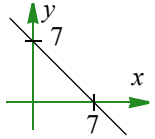
c



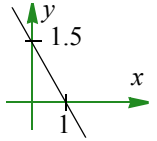
7 **a**



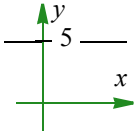
b



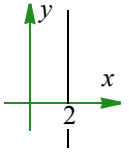
c



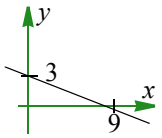
8 **a**



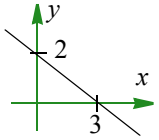
b



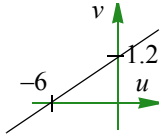
c



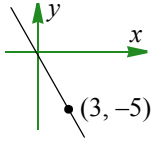
9 **a**



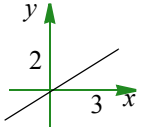
b



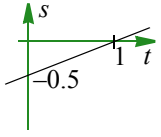
c



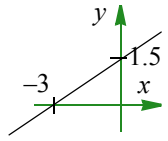
10 **a**



b



c

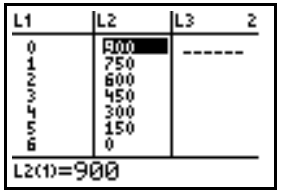


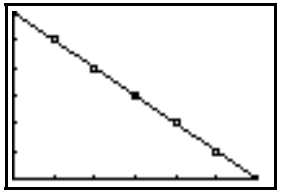
11 Check your graphs against the graphs with equations: **a** $y = 2x + 5$ **b** $y = -x + 3$ **c** $y = -2x + 8$ **d** $y = -0.75x + 3$ **e** $y = 2x - 2$

12 **a**

t (years)	0	1	2	3	4	5	6
V ($)	900	750	600	450	300	150	0

b



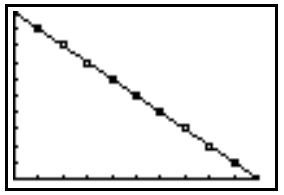


c Value of computer ≥ $0

13 **a**

t (hours)	0	1	2	3	4	5	6	7	8	9	10
V (litres)	250	225	200	175	150	125	100	75	50	25	0

14 **b** **c**

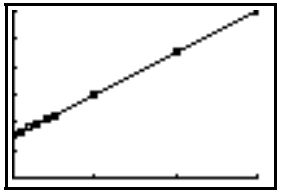


d $V \geq 0, 0 \leq t \leq 10$

15 **a**

n	0	1	2	3	4	5	10	20	30	40	...
I	750	825	900	975	1050	1125	1500	2250	3000	3750	...

b

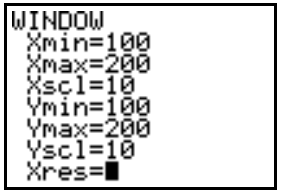


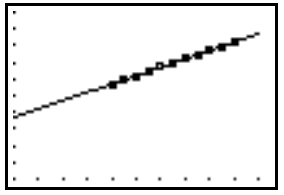
c Same as **b**. Note: this is a discrete linear relation.

16 **a** An appropriate range for the father's height would be:

H	140	145	150	155	160	165	170	175	180	185	190
h	157	159.5	162	164.5	167	169.5	172	174.5	177	179.5	182

b





c 180.5 cm

Exercise 6.2

1 **a** 1.5 **b** 1 **c** 2 **d** $\frac{8}{3}$ **e** $-\frac{9}{7}$ **f** 0.6 **g** –0.25 **h** 3 **i** a **j** –2

2 a 1.5 **b** –0.75 **c** 2.5 **d** 1.25 **e** 0 **f** undefined

3 **a** $y = 2x + 5$ **b** $y = -x + 3$ **c** $y = -2x + 8$ **d** $y = -0.5x + 3$ **e** $3y = 2x$

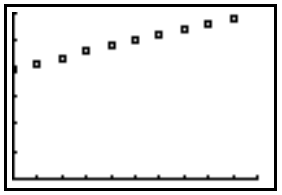
4 **a** $y = -\frac{3}{4}x + 3$ **b** $y = 2x - 2$ **c** $y = 2x - 5$ **d** $y = -x - 4$ **e** $y = -3x + 10$

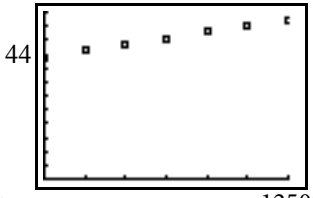
5 **a** $2y = 3x$ **b** $3x + 4y = 12$ **c** $2y - 5x + 10 = 0$ **d** $4y - 5x = 20$ **e** $y = 3$ **f** $x = 2$

6 Straight line should be drawn in such a way that it passes through the two intercepts: **a** *x-int*: (4, 0), *y-int*: (0, 2) **b** *x-int*: (–6, 0), *y-int*: (0, 2)

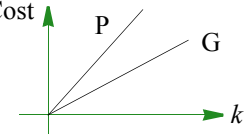
c *x-int*: (−2.25, 0), *y-int*: (0, 9) **d** *x-int*: (−2, 0), *y-int*: (0, 2)
e *x-int*: (−2, 0), *y-int*: (0, 3) **f** *x-int*: (−1, 0), *y-int*: (0, −2.5)

7 Straight line should be drawn in such a way that it passes through the two intercepts: **a** *x-int*: (2, 0), *y-int*: $\left(0, -\frac{4}{3}\right)$ **b** *x-int*: (−10, 0), *y-int*: (0, −9)
c *x-int*: (7, 0), *y-int*: (0, 14)

8 **a** $C = 4 + 0.02n$, $n = 0, 1, 2, 3\ldots$ **b**  **c** \$44

9 **a** $B = 44 + 0.22n$, $n = 0, 1, 2, \ldots$ **b** 

c \$80.96 **d** 192

10 **a i** $P = 0.35k$ **ii** $G = 0.27k$ **b** 

c 12500 km

Exercise 6.3.1

1 **a** (3, 3) **b** (2, −1) **c** (−1.5, −1.5) **d** (−1, 1)

2 **a** (2, 2) **b** $\left(\frac{1}{3}, \frac{5}{3}\right)$ **c** (−10, 8)

3 **a** $\left(\frac{5}{2}, \frac{15}{2}\right)$ **b** $\left(\frac{9}{5}, \frac{51}{5}\right)$ **c** (−3, 1)

4 **a** $x = -3, y = -0.5$ **b** $u = -0.75, v = 4.25$ **c** $m = -1, n = 5$ **d** $a = 2, b = -2$
e no solution **f** no solution

5 **a** $x = 1, y = 4$ **b** $x = 3, y = 2$ **c** $x = 2, y = -1$ **d** $x = 4, y = 5$ **e** $x = 4, y = -3$
f $x = -3, y = 2$

Exercise 6.3.2

1 **a** $x = 1, y = 2$ **b** $x = 3, y = 5$ **c** $x = -1, y = 2$ **d** $x = 0, y = 1$ **e** $x = -2, y = -3$
f $x = -5, y = 1$

2 **a** $x = \frac{13}{11}, y = \frac{17}{11}$ **b** $x = \frac{9}{14}, y = \frac{3}{14}$ **c** $x = 0, y = 0$ **d** $x = \frac{4}{17}, y = -\frac{22}{17}$
e $x = -\frac{16}{7}, y = \frac{78}{7}$ **f** $x = \frac{5}{42}, y = -\frac{3}{28}$

3 **a** (3, −3) **b** (2, −1) **c** (3, 1) **d** (3, −1) **e** (−1, −4) **f** $\left(\frac{62}{15}, \frac{19}{15}\right)$

4 **a** $x = a, y = 0$ **b** $x = \frac{1}{2}(a+b), y = \frac{1}{2}(a-b)$ **c** $x = \frac{a}{2}, y = -\frac{a}{2}$

5 **a** Lines are parallel with different *y*-intercepts, so no solution. **b** Lines are parallel with different *y*-intercepts, so no solution. **c** Lines are parallel with same *y*-intercepts (i.e. they are coincident), so there are an infinite number of solutions.

6 **a** −3 **b** −5 **c** −1.5

7 **a** $m = 2, a = 8$ **b** $m = 10, a = 24$ **c** $m = -6, a = 9$

8 **a** $a = 1, b = 1$ **b** $a = \frac{9}{4}, b = 3$

9 **a** $\{(a^{-1}, 0)\}$ **b** $\{(a, b)\}$ **c** $\{(b, 0)\}$ **d** $\{(1, 0)\}$

Exercise 6.3.3

1 CD: \$8.50 video: \$6.00
2 shirt: \$9.00 socks: \$2.00
3 length 15 m, width 12 m
4 maths 72, physics 44
5 adult \$5.00, child \$2.00
6 brown \$10, yellow \$12
7 21600
8 \$26 (the tapes cost \$18).
9 Adults \$17, children \$9.
10 **a** $x = 48, y = -70$ **b** $x = 94.67, y = -140$ **c** Graphically, the lines are almost parallel so small changes in the coefficients produce big changes in the solutions.

Exercise 6.4 Miscellaneous questions

1 **a** (2, −1) **b** (3, 4) **c** $\left(-4, -\frac{11}{2}\right)$ **d** (2, 3)

2 **a** $r = \frac{16}{5}, s = \frac{2}{5}$ **b** (3, 1) **c** (−3, 2) **d** (2, 7)

3 **a** (3, −1) **b** (16, 14) **c** $\left(\frac{18}{5}, \frac{3}{5}\right)$ **d** $u = k, v = 4 - 4k, k \in \mathbb{R}$ *(i.e. infinite soln).*

4 **a** (−2, 1) **b** $p = 0.5, q = 4$ **c** (0, 3) **d** $u = 0.5, v = 1.5$ **e** $m = \frac{3}{14}, n = \frac{11}{28}$

5 **a** $\left(\frac{a+b}{2a}, \frac{a-b}{2}\right)$ **b** $p = \frac{(a-1)b}{b-a}, q = \frac{a^2-b}{b-a}$

6 $\left(\frac{2ab}{a+b}, \frac{2ab}{a+b}\right)$

7 36, 21
8 12, 9
9 35, 7
10 \$6.20
11 7 two-cent coins, 32 one-cent coins
12 $i = 5, j = 1$
13 $p = 1.36, q = 1.28$
14 \$5.00
15 $a = 7, b = 3$

16 Felicity is 36, Joanne is 43
17 4.5 kmh^{-1} and 3.75 kmh^{-1}
18 72
19 $A = 336\ \text{m}^2$ dim: 12×28
20 **a** 0.75 km h^{-1} **b** 6.75 km h^{-1}

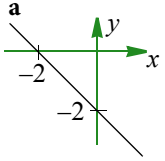
6.5 Graded revision questions

LEVEL 1

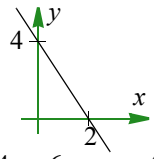
1 **a** $m = 2, c = 5$ **b** $m = -0.3, c = -0.4$ **c** $m = -0.4, c = 1.6$
2 **a** (0, 2) **b** (−1.5, −1.5) **c** (−3, 4)
3 **a** 2.2 **b** −2.4 **c** −4

LEVEL 2

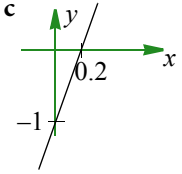
1 **a** −4.5 **b** 2
2 (0, 10)
3 **a**



b



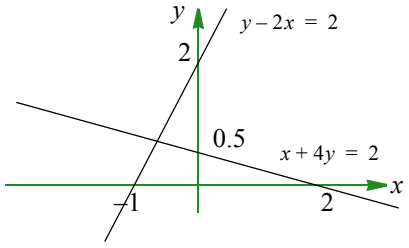
c



4 **a** $2y - x = 1$ **b** $3y + 4x = 6$ **c** $y = 2$
5 150
6 $(a, 0)$

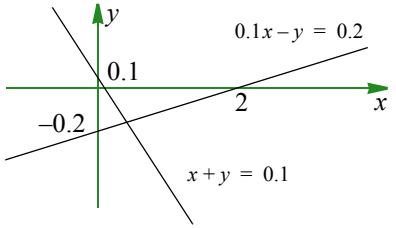
LEVEL 3

1 **a** $x = 3, y = 4$ **b** $x = 3, y = 5$ **c** $x = 13, y = 5$
2 **a** $2y + 9x = 41$ **b** $y + x = 1$ **c** $y - x = 2b$
3 **a i**



ii $\left(-\frac{2}{3}, \frac{2}{3}\right)$

b i



ii $\left(\frac{3}{11}, -\frac{19}{110}\right)$

4 $\sqrt{41}$
5 −13.5
6 \$2

LEVEL 4

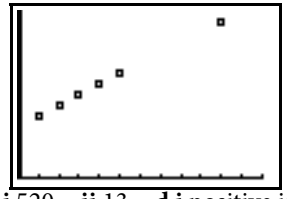
1 $a = \frac{16}{7}$, $b = \frac{20}{7}$
2 $x = a$, but no finite y.
3 $x = -2, y = 1, z = 4$
4 **a i** $\left(\frac{k}{2}, 0\right)$ **ii** $\left(0, \frac{k}{3}\right)$ **b** need to sketch graph **c** $k = 6$

6.6 Topic test

1 $m = -3$, (0, 7)
2 $m = 2$
3 **a** $y = -\frac{2}{3}x + 2$ **b** $a = -\frac{3}{2}$
4 (3, 3)
5 $\left(\frac{5}{4}, \frac{7}{4}\right)$
6 adult \$40, teen \$8
7 **a**

n	0	1	2	3	4	5	10
Total income (\$$T$)	200	240	280	320	360	400	600

b



c i 520 **ii** 13 **d i** positive integers including zero **ii** $T = 200 + 40n$, $n = 0, 1, 2, \ldots$

Exercise 7.1

1 **a** $2(ax + 2)$ **b** $y(9 - a)$ **c** $t(4 + s)$ **d** $x^2(1 + x)$ **e** $xy(3 - y)$ **f** $rs(r^2 + s^2)$
g $z(2z - 3y)$ **h** $ab^2(1 - a^3b)$ **i** $xy(3xy - 8)$ **j** $3y^2(xy + 3)$ **k** $2wt(1 + 4w)$
l $3ps(1 - 4ps)$
2 **a** $5a(x + 2y - xy)$ **b** $xu(2x - 4u + 1)$ **c** $mn(3 + 4n)$ **d** $ab^2(c - a^2 + abc)$
e $3xy(y^2 + 2x - 4)$ **f** $3uv(4 - 3u - 2uv)$
3 **a** $(2 + z)(x + y)$ **b** $(3 - r)(t - s)$ **c** $(x - y)(x + 2)$ **d** $(st + 2)(a - b)$
e $(r^2 + 4)(xy + 1)$ **f** $(y^3 + 5)(2 + z)$
4 **a** $(4 + x)(s + 1)$ **b** $(a + b)(2 - c)$ **c** $(2 + t)(5x + y)$ **d** $(3 - 5k)(x + 2)$
e $(b + 4)(ab - 1)$ **f** $(y - z)(a + b)$
5 **a** $(a^3 + 4)(a - 1)$ **b** $(1 - yz)(x + 1)$ **c** $(m - 1)(n - 1)$ **d** $(y^2 + 3)(y - 1)$
e $(x - b)(x - b + 1)$ **f** $(a + b)(a + b - 1)$ **g** $(2 - x)(a - b)$ **h** $(z + 3)(x - y)$

Exercise 7.2.1

1 **a** $(x+8)(x+1)$ **b** $(x+10)(x+1)$ **c** $(x+12)(x+1)$ **d** $(x+6)(x+4)$
e $(x+3)(x+5)$ **f** $(x+9)(x+4)$ **g** $(x+8)(x+2)$ **h** $(x+8)(x+3)$
i $(x+5)(x+5) = (x+5)^2$ **j** $(x+6)(x+6) = (x+6)^2$ **k** $(x+11)(x+3)$
l $(x+13)(x+6)$

2 **a** $(x-8)(x-1)$ **b** $(x-10)(x-1)$ **c** $(x-12)(x-1)$ **d** $(x-8)(x-2)$
e $(x-5)(x-4)$ **f** $(x-8)(x-4)$ **g** $(x-9)(x-4)$ **h** $(x-10)(x-11)$
i $(x-7)(x-8)$ **j** $(x-9)(x-3)$ **k** $(x-7)(x-5)$ **l** $(x-6)(x-6) = (x-6)^2$

3 **a** $(x+7)(x-4)$ **b** $(x+9)(x-3)$ **c** $(x+6)(x-2)$ **d** $(x+12)(x-4)$
e $(x+8)(x-5)$ **f** $(x+12)(x-6)$ **g** $(x+14)(x-3)$ **h** $(x+15)(x-5)$
i $(x+11)(x-9)$ **j** $(x+9)(x-8)$ **k** $(x+13)(x-3)$ **l** $(x+13)(x-4)$

4 **a** $(x+3)(x-7)$ **b)** $(x-8)(x+2)$ **c** $(x-9)(x+4)$ **d** $(x-12)(x+2)$
e $(x-14)(x+4)$ **f** $(x-12)(x+6)$ **g** $(x-15)(x+5)$ **h** $(x-15)(x+7)$
i $(x-13)(x+4)$ **j** $(x-13)(x+7)$ **k** $(x-16)(x+4)$ **l** $(x-7)(x+6)$

Exercise 7.2.2

1 **a** $(2x+1)(x+1)$ **b** $(2x+1)(x+2)$ **c** $(2x+1)(x+3)$ **d** $(3x+1)(x+1)$
e $(3x+1)(x+2)$ **f** $(3x+1)(x+3)$

2 **a** $(2x-1)(x-1)$ **b** $(2x-1)(x-2)$ **c** $(2x-1)(x-3)$ **d** $(3x-1)(x-1)$
e $(3x-1)(x-2)$ **f** $(3x-1)(x-3)$

3 **a** $(2x+1)(x-1)$ **b** $(2x+1)(x-2)$ **c** $(2x+1)(x-3)$ **d** $(3x+1)(x-1)$
e $(3x+1)(x-2)$ **f** $(3x+1)(x-3)$ **g** $(2x+1)(3x-1)$ **h** $(2x-1)(3x+1)$
i $(2x-1)(2x-1) = (2x-1)^2$

4 **a** $-3(x+2)(x-1)$ **b** $3(x+2)(x+1)$ **c** $(5x-2)(x-1)$ **d** $(5x+2)(x-3)$
e $2(1-2x)(x+3)$ **f** $-(3x-1)(x+4)$ **g** $3(3x+2)(x+1)$ **h** $2(3-x)(x+2)$
i $(5x+2)(2x+5)$ **j** $(2x+1)(4x-3)$ **k** $(2-3x)(2x+3)$ **l** $(9x+1)(2-x)$

Exercise 7.2.3

1 **a** $(x+2)^2$ **b** $(x+7)^2$ **c** $(y+8)^2$ **d)** $(4x+1)^2$ **e** $(5a+1)^2$ **f** $(6z+1)^2$
g $(2x+3)^2$ **h** $(3x+4)^2$ **i** $(5x+2)^2$ **j** $(x+1)^2$ **k** $\left(x+\frac{1}{2}\right)^2$ **l** $\left(x+\frac{3}{2}\right)^2$

2 **a** $(x-2)^2$ **b** $(x-7)^2$ **c** $(y-8)^2$ **d** $(2u-3)^2$ **e** $(3a-4)^2$ **f** $(5v-2)^2$
g $\left(y-\frac{1}{2}\right)^2$ **h** $\left(a-\frac{5}{2}\right)^2$ **i** $\left(x-\frac{7}{2}\right)^2$

3 **a** $(a-y)(a+y)$ **b** $(y-1)(y+1)$ **c** $(10-a)(10+a)$ **d** $(8-b)(8+b)$ **e** $(9-z)(9+z)$
f $(r-7)(r+7)$ **g** $(4x-7)(4x+7)$ **h** $(5-2a)(5+2a)$ **i** $(1-3y)(1+3y)$
j $4(a-3)(a+3)$ **k** $2(m-2)(m+2)$ **l** $3(y-3)(y+3)$ **m** $(x+5)(x-1)$
n $(ab-a-b)(ab+a+b)$ **o** $-3y(2x-y)$

4 **a** $(x-\sqrt{3})(x+\sqrt{3})$ **b** $(y-\sqrt{5})(y+\sqrt{5})$ **c** $(b-\sqrt{7})(b+\sqrt{7})$ **d** $(a+b-\sqrt{5})(a+b+\sqrt{5})$
e $(a-b\sqrt{10})(a+b\sqrt{10})$ **f** $(x+y-\sqrt{2})(x+y+\sqrt{2})$ **g** $2(x+\sqrt{2}(x-y))(x-\sqrt{2}(x-y))$
h $4(x-\sqrt{5})(x+\sqrt{5})$ **i** $18(\sqrt{2}a-b)(\sqrt{2}a+b)$ **j** $(a+2-\sqrt{10})(a+2+\sqrt{10})$
k $(x-3-\sqrt{6})(x-3+\sqrt{6})$ **l** $(x-y)(2-x+y)(2+x-y)$

Exercise 7.2.4

1 **a** $(x+1+\sqrt{3})(x+1-\sqrt{3})$ **b** $(x+2-\sqrt{6})(x+2+\sqrt{6})$ **c** $(x-3-\sqrt{7})(x-3+\sqrt{7})$
d $(x-4-\sqrt{19})(x-4+\sqrt{19})$ **e** $(x-1-\sqrt{2})(x-1+\sqrt{2})$ **f** $(x-2-2\sqrt{2})(x-2+2\sqrt{2})$
g $2\left(x-\frac{3}{2}+\frac{\sqrt{13}}{2}\right)\left(x-\frac{3}{2}-\frac{\sqrt{13}}{2}\right)$ **h** $3(x+2+\sqrt{7})(x+2-\sqrt{7})$ **i** $2\left(x+\frac{1}{4}+\frac{\sqrt{17}}{4}\right)\left(x+\frac{1}{4}-\frac{\sqrt{17}}{4}\right)$

2 **a** $(x+1)(x+2)$ **b** $(x+6)(x+1)$ **c** $(x+4)(x+2)$ **d** $(x+4)(x+5)$
e $(z+6)(z+3)$ **f** $(x+2)(x+5)$

3 **a** $(x-2)(x-1)$ **b** $(x-6)(x-1)$ **c** $(x-4)(x-2)$ **d** $(x-4)(x-5)$ **e** $(z-6)(z-3)$
f $(x-2)(x-5)$

4 **a** $(x-3)(x+1)$ **b** $(y+5)(y-2)$ **c** $(s-5)(s+2)$ **d** $(x-4)(x+3)$ **e** $(y-7)(y+2)$
f $(r-9)(r+5)$

5 **a** $2(x-2)(x-1)$ **b** $3(x+2)(x+5)$ **c** $4(s-5)(s+2)$ **d** $3(y+5)(y-2)$
e $5(x+4)(x+2)$ **f** $6(x+1)(x-3)$

6 **a** $(y-15x)(y-x)$ **b** $(z+7w)(z-6w)$ **c** $(a+7b)(a-5b)$ **d** $2(y+6x)(y+3x)$
e $3(x-4y)(x+3y)$ **f** $5(a-2b)(a-3b)$

7 **a** $(2x+1)(x+2)$ **b** $(3x+2)(x+1)$ **c** $(3x+1)(x+2)$ **d** $(2x-1)(x-2)$
e $(2x-1)(x+2)$ **f** $(7s-5)(s+1)$ **g** $(5x-3)(x+1)$ **h** $(7x-1)(x-5)$
i $(3y+1)(y-2)$ **j** $(3z-4)(z-2)$ **k** $(5w+4)(w-1)$ **l** $(2x+3)(x-5)$
m $(2y+3)(y+2)$ **n** $(5x+4)(x+1)$ **o** $(3z+2)(z+2)$ **p** $2(1-4x)(1+x)$
q $(3-5x)(1+x)$ **r** $(4+5x)(1-x)$

8 **a** $(x+4)^2$ **b** $(y+5)^2$ **c** $(z+3)^2$ **d** $(x-5)^2$ **e** $(b-6)^2$ **f** $(x-7)^2$
g $(y-5)(y+5)$ **h** $(x-6)(x+6)$ **i** $(z-4)(z+4)$ **j** $4x(x-3)(x+3)$
k $3(s-4)(s+4)$ **l** $(2x-3y)(2x+3y)$ **m** $(2x+1)^2$ **n** $(3z+2)^2$ **o** $(3z+1)^2$

9 **a** $(x+1-\sqrt{5})(x+1+\sqrt{5})$ **b** $(x-1-\sqrt{3})(x-1+\sqrt{3})$
c $(x+2-\sqrt{2})(x+2+\sqrt{2})$ **d** $(x+2-\sqrt{7})(x+2+\sqrt{7})$ **e** $(x-3-\sqrt{6})(x-3+\sqrt{6})$ **f** $(x+3-\sqrt{7})(x+3+\sqrt{7})$ **g** $(x+4-\sqrt{3})(x+4+\sqrt{3})$
h $(x-4-\sqrt{2})(x-4+\sqrt{2})$ **i** $(z+5-\sqrt{5})(z+5+\sqrt{5})$ **j** $(x-5-\sqrt{5})(x-5+\sqrt{5})$
k $(z-2-\sqrt{3})(z-2+\sqrt{3})$ **l** $(a+7-\sqrt{19})(a+7+\sqrt{19})$

Exercise 7.3.1

1 **a** 0, 7 **b** $\pm\sqrt{7}$ **c** 0, –3 **d** no real solutions **e** –4, 3 **f** 0, 2 **g** –4, 2 **h** –2, 5
i –2, 5 **j** –6, 5 **k** –6, 1 **l** –4, 7 **m** –4, 3.5 **n** 4, 7 **o** 3, 9 **p** –3, 9
q –4, –3 **r** –3, –4/3 **s** –7, –2 **t** –0.5, 4 **u** –1/3, 4 **v** 0.5, 5 **w** 0.5, 5/3
x 1/4, 5/3 **y** 1/3, 2/5 **z** 1/4, 2

Exercise 7.3.2

1 **a** $\frac{-5\pm\sqrt{17}}{2}$ **b** $-2\pm\sqrt{7}$ **c** $1\pm\sqrt{6}$ **d** no real solutions **e** $\frac{-5\pm\sqrt{53}}{2}$ **f** $\frac{-7\pm\sqrt{85}}{2}$
g $\frac{-11\pm\sqrt{157}}{2}$ **h** $\frac{-3\pm\sqrt{57}}{2}$ **i** $\frac{-5\pm\sqrt{73}}{2}$ **j** no real solutions **k** $2\pm\sqrt{11}$ **l** $3\pm2\sqrt{5}$

m $-2 \pm 2\sqrt{6}$ **n** no real solutions **o** $1 \pm \frac{\sqrt{22}}{2}$ **p** $-\frac{1}{2}, 3$ **q** $-\frac{1}{2}, 4$ **r** $\frac{3 \pm \sqrt{41}}{4}$

s $\frac{3 \pm \sqrt{57}}{6}$ **t** $\frac{5 \pm \sqrt{73}}{6}$

Exercise 7.3.3

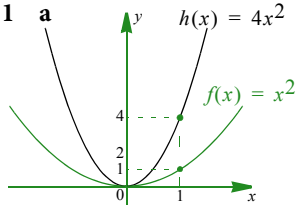
1 **a** $\frac{3 \pm \sqrt{37}}{2}$ **b** $\frac{5 \pm \sqrt{33}}{2}$ **c** $\frac{3 \pm \sqrt{33}}{2}$ **d** $\frac{7 \pm \sqrt{57}}{2}$ **e** $\frac{-7 \pm \sqrt{65}}{2}$ **f** $-4, 2$ **g** $-1 \pm 2\sqrt{2}$

2 **h** $\frac{-5 \pm \sqrt{53}}{2}$ **i** $\frac{3 \pm \sqrt{37}}{2}$ **j** no real solutions **k** $4 \pm \sqrt{7}$ **l** no real solutions

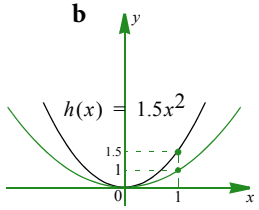
m $\frac{2 \pm \sqrt{13}}{2}$ **n** $\frac{3 \pm 2\sqrt{11}}{5}$ **o** $\frac{6 \pm \sqrt{31}}{5}$ **p** $\frac{6 \pm \sqrt{29}}{7}$

Exercise 7.4.1

1 **a**



b



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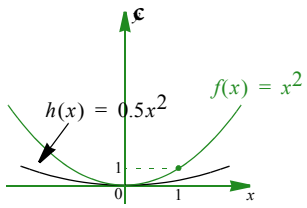

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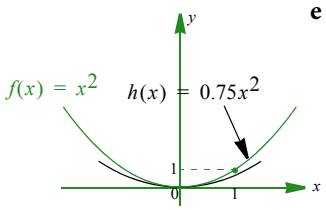

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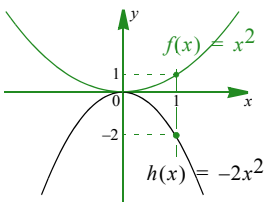
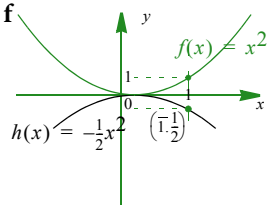
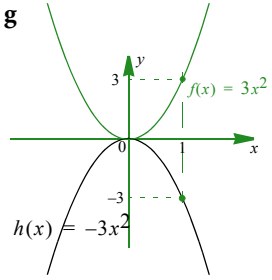


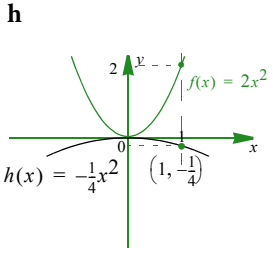
f



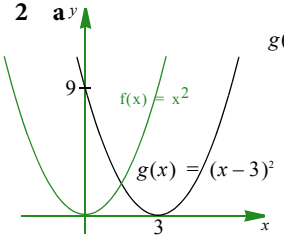
g



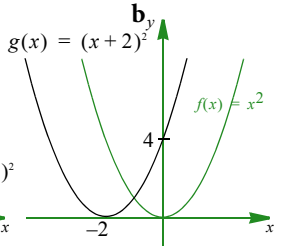
h



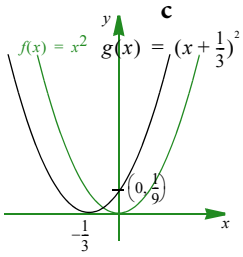
2 **a**



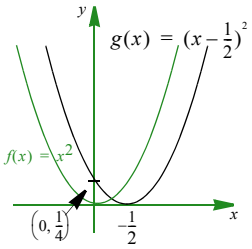
b



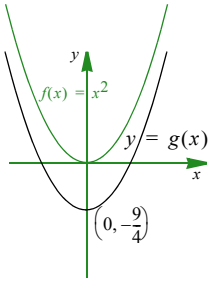
c



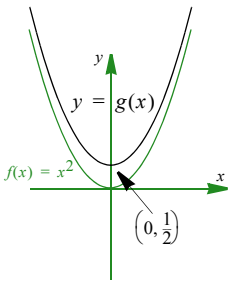
d



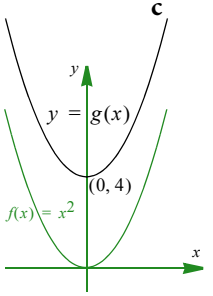
3 **a**



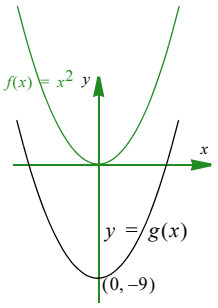
b



c

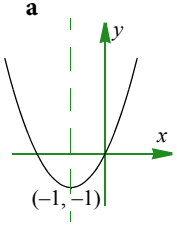


d

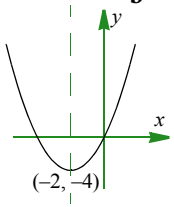


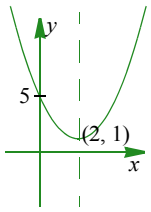
Exercise 7.4.2

1 a

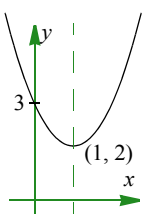


b

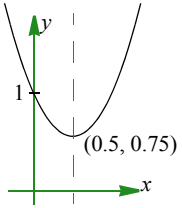




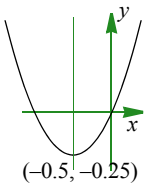
cd



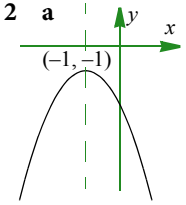
e



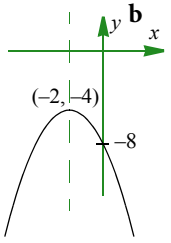
f

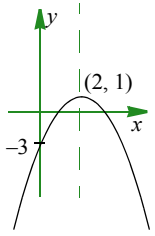


2 a

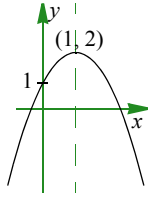


b

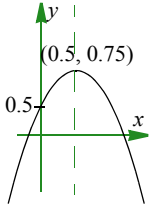




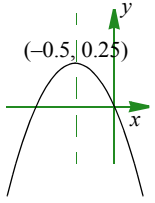
c d



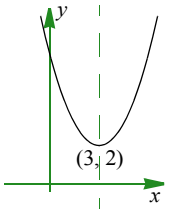
e



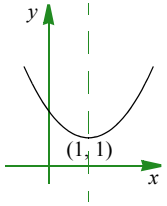
f

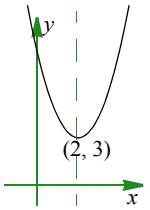


3 a

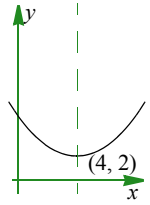


b

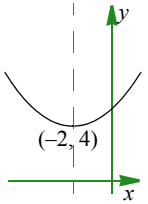




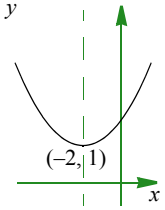
cd



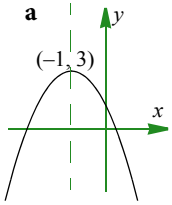
e



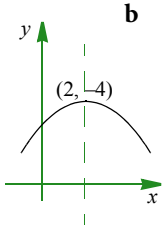
f

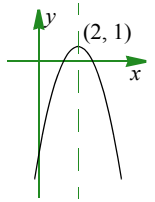


4 a

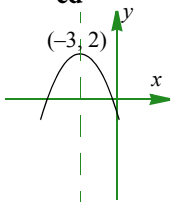


b

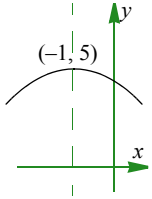




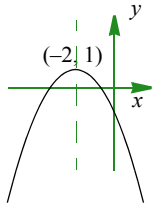
cd



e



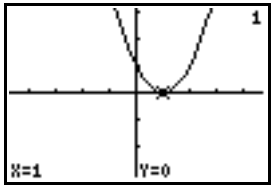
f



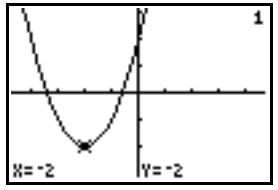
Exercise 7.4.3

1 Graphs are shown using the **ZOOM4** viewing window:

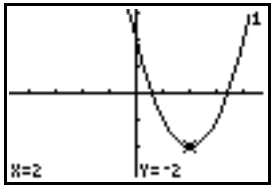
a

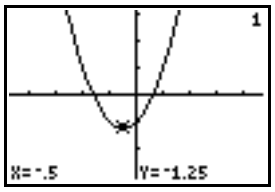


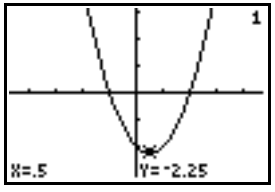
b

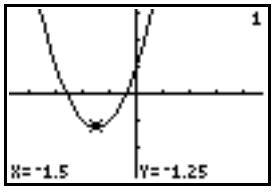


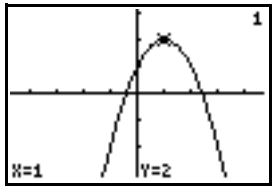
c

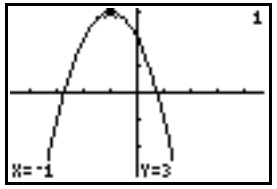


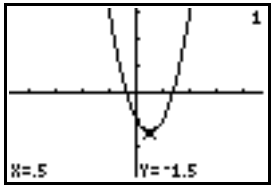
d 

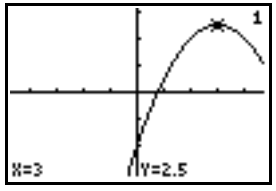
e 

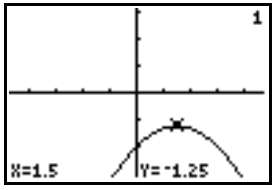
f 

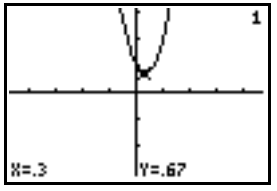
g 

h 

i 

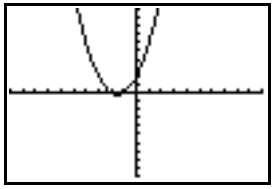
j 

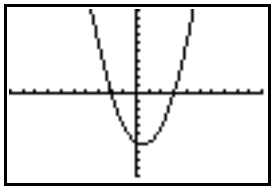
k 

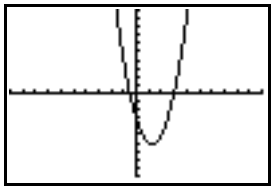
l 

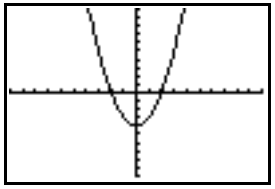
Exercise 7.4.4

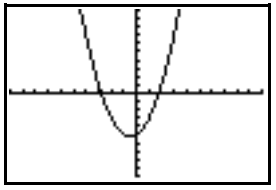
1 Graphs are shown using the **ZOOM6** viewing window:

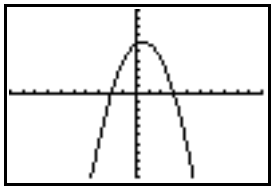
a 
(–2, 0), (–1, 0), (0, 2)

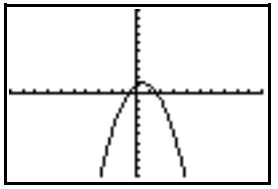
b 
(–2, 0), (3, 0), (0, –6)

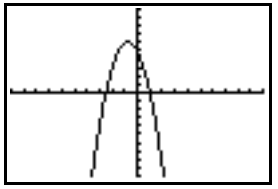
c 
(–0.5, 0), (3, 0), (0, –3)

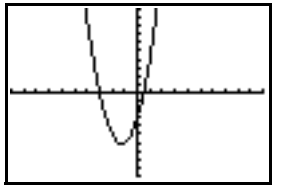
d 
(–2, 0), (2, 0), (0, –4)

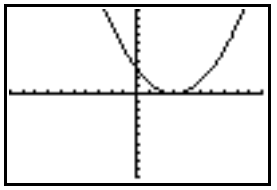
e 
(–2.79, 0), (1.79, 0), (0, –5)

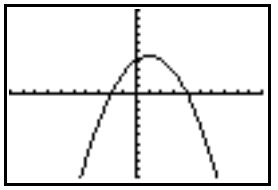
f 
(–2, 0), (3, 0), (0, 6)

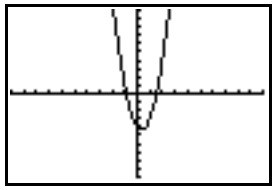
g 
(–0.62, 0), (1.62, 0), (0, 1)

h 
(–2.5, 0), (1, 0), (0, 5)

i 
(–3, 0), (0.5, 0), (0, –3)

j 
(3, 0), (0, 3)

k 
(–2, 0), (4, 0), (0, 4)

l 
(–0.87, 0), (1.54, 0), (0, –4)

Exercise 7.5

1 **a** $(x-2)(x-1)$ **b** $(x-6)(x-1)$ **c** $(x-4)(x-2)$ **d** $(x-4)(x-5)$
e $(z-6)(z-3)$ **f** $(x-2)(x-5)$ **g** $(x-3)(x+1)$ **h** $(y+5)(y-2)$
i $(s-5)(s+2)$ **j** $(x-4)(x+3)$ **k** $(y-7)(y+2)$ **l** $(r-9)(r+5)$
m $2(x-1)(x-2)$ **n** $3(x+5)(x+2)$ **o** $4(s-5)(s+2)$ **p** $3(y+5)(y-2)$
q $5(x+4)(x+2)$ **r** $6(x-3)(x+1)$

Exercise 7.6.1

1 $x = 1, 5$
2 $x = -2, 3$
3 $x = -2, 2$
4 $x = -1, 3$
5 $x = -2, 2$
6 **a** (–2, 4); (3, 9) **b** (2, 5); (–2, 5) **c** (1, 2) **d** (–2, 0); (3, 5)
e (–3, 7); (2, 2) **f** (3, 9)
7 $(a, 3a^2)$; $(-2a, -3a^2)$

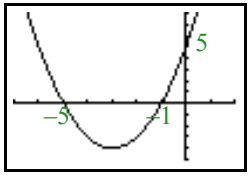
Exercise 7.6.2

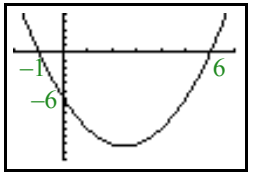
1 **b** (–2, 4); (2, 4)
2 (–1, 2); (1, 2)
3 (–1, 3); (2, 0)
4 **a** (3, 17); (–2, 2) **b** (1, 6); (2, 13) **c** (–1, 1); (1, 9) **d** (0.5, 2.25); (–2, 1)
e (4, 46); (5, 69) **f** (–3, 29); (–2, 12)

Exercise 7.7 Miscellaneous questions

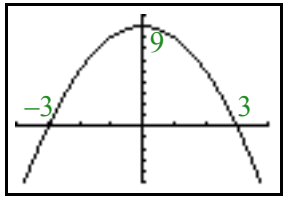
1 **a** $4y(x-3y)$ **b** $2ab(3a+2b)$ **c** $3uv(v-4u)(v+u)$
2 23 units
3 (2, –16)
4 none
5 $x = 1, (1, -1)$

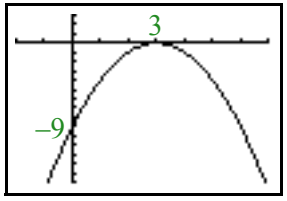
6 25
7 **a** 2 **b** 4
8 −21
9 −2
10 **a** $(3+y)(x-4)$ **b** $x=4$, $y=-3$
11 75
12 2
13 72
14 4, 6
15 −1.5, 11
16 **a** $x=1.5$ **b** none **c** $x=0$ or $x=6$ **d** all real values
17 2
18 **a** −8, 3 **b** $\frac{5+\sqrt{57}}{4}, \frac{5-\sqrt{57}}{4}$
19 $b=0$, $c=-7$
20 2
21 **a** $(x+4)(x-2)$ **b** Using the graph of $y=(x+4)(x-2)$ we have $x=-3, -2, -1, 0, 1$.
22 **a i** $(x+5)(x+1)$ **ii** $(x-6)(x+1)$ **b i** **ii**



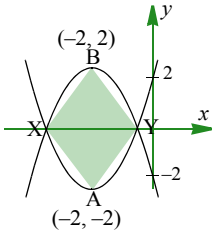


23 **a** $(x-4)(x+3)$ **b** $(3x-2)(2x-3)$ **c** $(x-1-y)(x-1+y)$
24 **a** **b**





25 **a** $b=16$, $a=10$ **b** $(x-8)(x-2)$
26 $y=\left(x-\frac{3}{2}\right)^2+4\frac{3}{4}$, $\left(\frac{3}{2}, 4\frac{3}{4}\right)$
27 $a=-1$, $b=-3$
28 −3, 0.5
29 $(0, 7)$, $(1\pm 2\sqrt{2}, 0)$
30 $b^2-4ac>0$, $b<-6$ or $b>6$
31 **a** $a=0.5$, $b=-3$, $c=-8$ **b** $\frac{1}{2}(x+2)(x-8)$ **c i** −1, 0, 1, 2, 3, 4, 5, 6, 7 **ii** −12.5
32 **a** $(-2, -6)$; $\left(\frac{2}{3}, \frac{26}{9}\right)$ **b** $\left(-\frac{1}{2}, \frac{15}{4}\right)$; $(2, 0)$ **c** $(2, -1)$; $(-1, 2)$
33 **a** X$(-2-\sqrt{2}, 0)$ and Y$(-2+\sqrt{2}, 0)$



b i 4 units **ii** 8 sq. units
34 mn

7.8 Graded revision questions

LEVEL 1

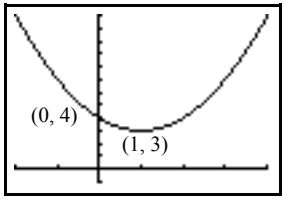
1 **a** No **b** Yes **c** Yes
2 **a** $x(x+1)$ **b** $x(2-x)$ **c** $ax(4+x)$ **d** $6y(y+2)$ **e** $3b(3b-1)$ **f** $4ay(3y+1)$
3 **a** $(2+a)(a+b)$ **b** $(z-3)(z-1)$ **c** $(x-2)(x+1)$

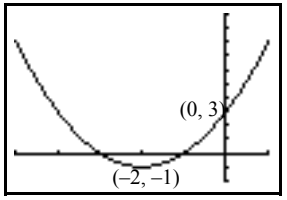
LEVEL 2

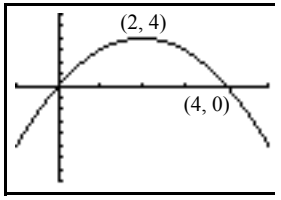
1 **a** $(x+4)(x-1)$ **b** $(y-5)(y-1)$ **c** $(x+7)(x-2)$ **d** $(a-1)^2$ **e** $(a-3)(a+1)$ **f** $(y-9)(y-7)$
2 **a** $(2x-1)(x+3)$ **b** $(3x+1)(x+1)$ **c** $(2y+3)(y-1)$ **d** $(3z+1)(2z+1)$ **e** $(4x+3)(3x+4)$ **f** $(5x+1)(x-3)$
3 **a** −4, 1 **b** 1, 5 **c** −3, −2 **d** 1 **e** −7, 2 **f** 7, 9
4 **a** $\frac{-1\pm\sqrt{29}}{2}$ **b** $\frac{-3\pm\sqrt{41}}{4}$ **c** $-2\pm\sqrt{13}$ **d** $\frac{3\pm\sqrt{89}}{10}$ **e** none **f** $\frac{-5\pm\sqrt{133}}{6}$

LEVEL 3

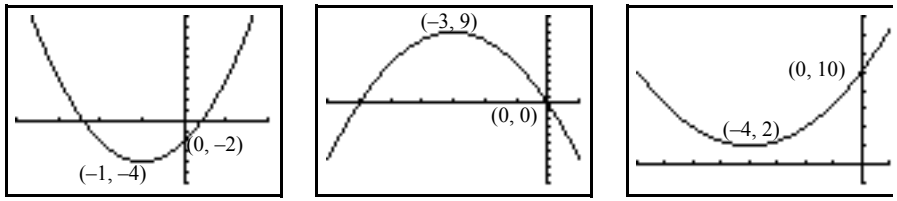
1 **a** (3, 4) **b** (−1, 3) **c** (1, −3) **d** (1, 1) **f** (0.5, 3) **g** (1, 0)
2 **a** $y=x^2-x-2$ **b** $y=4-x^2$ **c** $y=x^2-2x-1$ **d** $y=x^2-4x+4$
3 **a** $x=\frac{-1\pm\sqrt{17}}{2}$ **b** $x=\frac{1\pm\sqrt{15}}{2}$ **c** $x=3\pm\sqrt{10}$ **d** $x=2\pm\sqrt{3}$
e $x=\frac{1\pm\sqrt{17}}{4}$ **f** $x=3\pm\sqrt{7}$
4 **a** **b** **c**



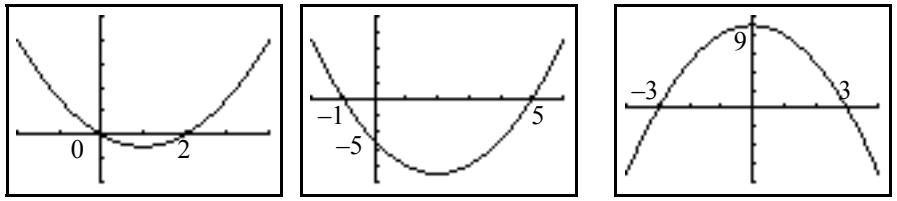




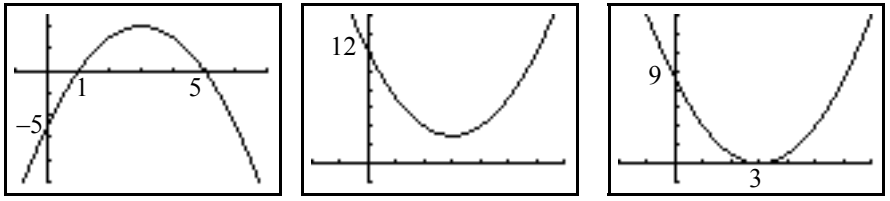
d **e** **f**



5 **a** **b** **c**



d **e** **f**



6 **a** (−5, 15); (3, 23) **b** (−4, −6); (2, 6)

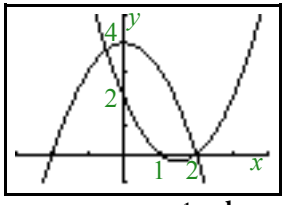
7 **a** $a = 2$, $k = 4$, $c = -4$ **b** −0.5, 4 **c** $x = 1.75$ **d** −2.125

LEVEL 4

1 $\left(-\frac{k}{2}, -\frac{k^2}{4} + 4\right)$

2 $N_{\min} = \frac{3}{4}$

3 **a**

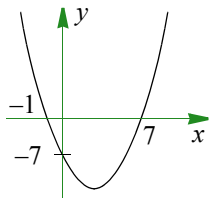


b $x = -0.5$ or $x = 2$ **c** (−0.5, 3.75) & (2, 0)

4 **a** $x = -a$ or $x = -1$ **b** $x = -2$ or $x = -1$.

7.9 Topic test

1 $x = 7$ or $x = -1$



2 **a** $(x-8)(x-4)$ **b** $(x-6)^2$ **c** $(x-6-\sqrt{6})(x-6+\sqrt{6})$

3 0.884, −1.88

4 $y = -x^2 + 2x + 3$, (1, 4)

5 −2, 3

6 $k = 3$

7 **a i** $-(x-10)(x-4)$ **ii** $x(8-x)$ A ≡ (4, 0), B ≡ (8, 0) and C ≡ (10, 0)

b $x = \frac{20}{3}$ **c** $\frac{80}{9}$sq.u

8 **a** $y = -0.5x + 1$ **b** $y = 2x(2-x)$ **c** (0.25, 0.875)

Exercise 8.1

1 **a** dom = {2, 3, −2}, ran = {4, −9, 9} **b** dom = {1, 2, 3, 5, 7, 9}, ran = {2, 3, 4, 6, 8, 10}
c dom = {0, 1}, ran = {1, 2}

2 **a**]1, ∞[**b** [0, ∞[**c**]9, ∞[**d**]−∞, 1] **e** [−3, 3] **f**]−∞, ∞[**g**]−1, 0] **h** [0, 4]
i [0, ∞[**j** [1, 5] **k**]0, 4[**l**]−∞, −1] ∪ [1, ∞[

3 **a** $r = [-1, \infty[$, $d = [0, 2[$ **b** $r = \{y : y \geq 0\} \setminus \{4\}$, $\mathbb{R}$ **c** $r = [0, \infty[\setminus \{3\}$, $d = [-4, \infty[\setminus \{0\}$
d $r = [-2, 0[$, $d = [-1, 2[$ **e** $r =]-\infty, \infty[$ $d =]-\infty, -3] \cup [3, \infty[$
f $r = [-4,4]$, $d = [0,8]$

4 **a** one to many **b** many to one **c** many to one **d** one to one
e many to many **f** one to one

5 **a** $\mathbb{R} \setminus \{-2\}$ **b**]−∞, 9[**c** [−4,4] **d**]−∞, −2] ∪ [2, ∞[**e** $\mathbb{R} \setminus \{0\}$ **f** $\mathbb{R}$

Exercise 8.2

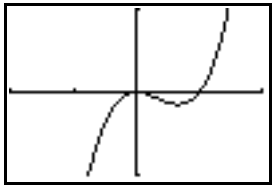
Graphs with graphics calculator output have standard viewing window unless otherwise stated.

1 **a** 3, 5 **b i** $2(x+a) + 3$ **ii** $2a$ **c** 3

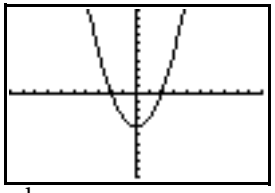
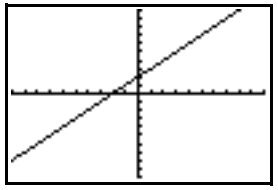
2 **a** 0, $\frac{10}{11}$ **b** $-\frac{5}{4}$ **c** $\left[0, \frac{10}{11}\right]$

3 **a** $-0.5x^2 - x + 1.5$, $-0.5x^2 + x + 1.5$ **b** $\pm\sqrt{2}$ **c** no solution

4 **a** $x = 0, 1$ **b**

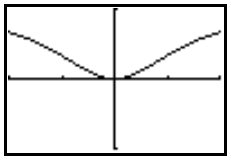


Window [−2,2], [−1,1]
Range: [−12, 4]

5 **a i**  **ii**  **b i** $\{2\sqrt{2}, -2\sqrt{2}\}$ **ii** $\{3, -2\}$

6 b, c, d, e

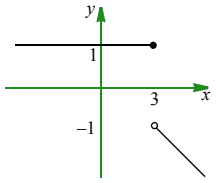
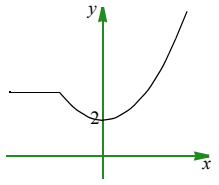
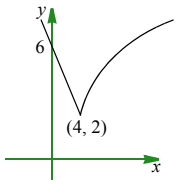
7 a, d, e, f

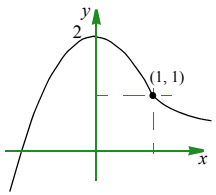
8 **a**  Window $[-2, 2]$, $[-1, 1]$ **b** $[0, 1[$

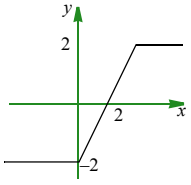
9 **a** $\{y : y > 1\} \cup \{y : y \leq -1.25\}$ **b** 10

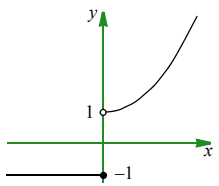
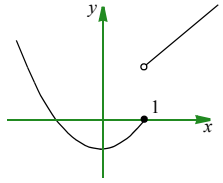
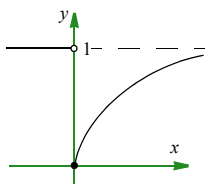
10 **a** $[-3, \infty[$ b $[-3, 0]$ c $[3, \infty[$

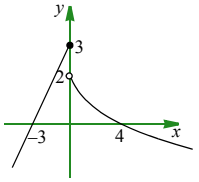
Exercise 8.3.1

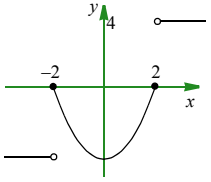
1 **a**  **b**  **c** 

d 

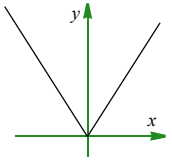
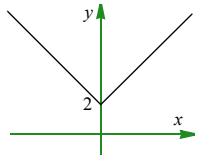
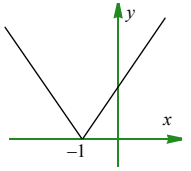
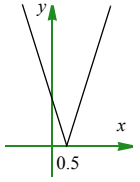
2 

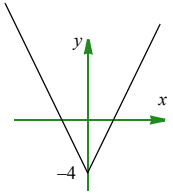
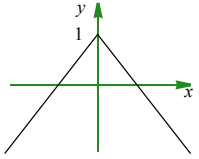
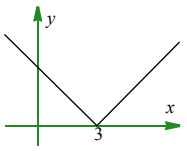
3 **a**  **b**  **c** 

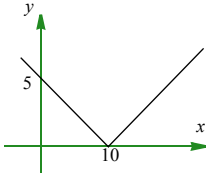
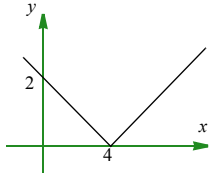
d 

4 

Exercise 8.3.2

1 **a**  **b**  **c**  **d** 

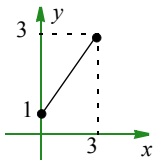
e  **f**  **g** 

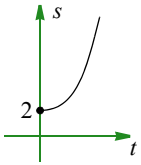
h  **i** 

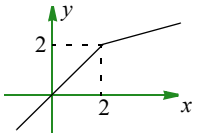
Exercise 8.4

1 **a** $-2, 1$ **b** $1, 2$ **c** 1.25 **d** 1.52 **e** 1 **f** 2.04 **g** $1.25, -1.25$ **h** 0.34, 8.99 **i** no solution **j** 3 **k** 3.44 **l** -1.67

Exercise 8.5 Miscellaneous questions

1 **a** Yes **b**  **c i** $0 \le x \le 3$ **ii** [1, 3] **d** None, $x = 12$ lies outside the domain

2 **a** 2 m **b** 66 m **c** 

3 

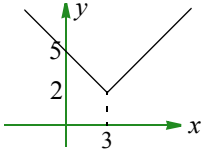
4 **a** $y \ge -2$ **b i** $(0, 4)$ **ii** $(3 \pm \sqrt{3}, 0)$ **c** $b = 2$, $a = \frac{2}{3}$

5 **a** 60, 44, 36, 32, 30 **b** Use graphics calculator **c** 28 **d** $\frac{16}{3}$ minutes

6 **a** 3, $a = 2, -1$

7 **b** $b = 3$, $c = 2$

8 Range $=]-\infty, 7]$; Many:one

9 

10 1

11 **b** $V = 0$ 60 64 36 0 **c** 67.6

8.6 Graded revision questions

LEVEL 1

1 **a** dom $= (-1, 4]$, ran $= [0, 5]$ **b** dom $= [0, 6]$, ran $= [0, 6]$
c dom $= \mathbb{R} \backslash \{2\}$, ran $= \mathbb{R} \backslash \{3\}$

2 a and c

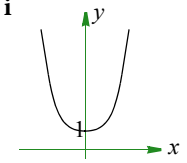
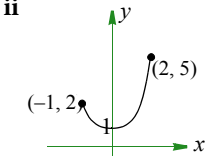
3 **a** $y = 2 + 2 = 4$ **b** 18 **c** 6

LEVEL 2

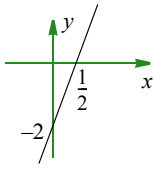
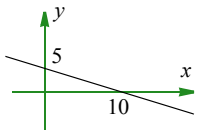
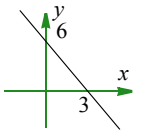
1 $-\frac{8}{3}$

2 **a** $x \le 2$ **b** $x \ge 3$ **c** $x > -1$

3 **a** $g(f(x)) = \frac{1}{2x}$ **b** $f(g(x)) = \frac{1-x}{1+x}$

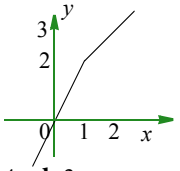
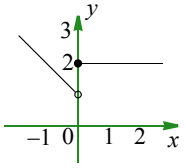
4 **a i**  **ii**  **b i** ran $= [1, \infty)$ **ii** ran $= [1, 5]$

5 **a** -1 **b** $4x^2 + 4x$ **c** $x^2 - 2x$ **d** $4x$

6 **a**  **b**  **c** 

7 **a i** 0 **ii** $h^2 - 3h$ **b** $3h + h^2$

LEVEL 3

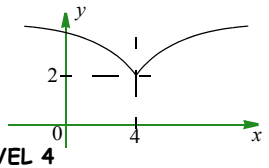
1 **a**  **b** 

2 **a** 4 **b** $3 + x$

3 $g(f(x)) = x$; $f(g(x)) = x$

4 Calculator check

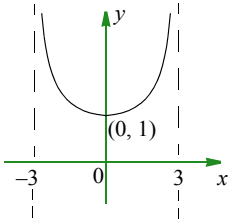
5 $x = \frac{k}{k-1}$

6 

LEVEL 4

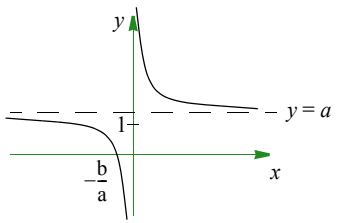
1 $S = (0, 2)$

2 $\{\pm\sqrt{3}, \pm 1\}$

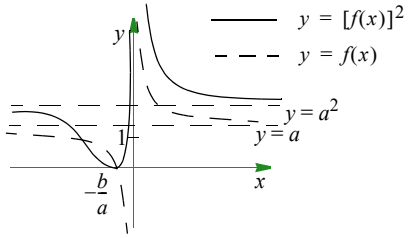
3 $-3 < x < 3$ 

4 **a** $\therefore a = 2, b = 8, c = 6$ **b** -0.75

5 a



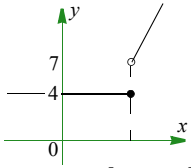
b i & ii



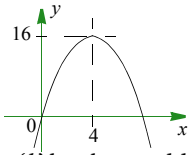
8.7 Topic test

1 a 1 **b** 1

2

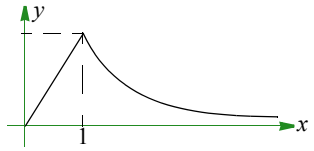


3 a range = $]-\infty, 16]$ **b** range = $]-\infty, 16 + k]$



This occurs because all that has happened is that 'k' has been added to the function in **a** and so the graph in **a** is moved up 'k' units.

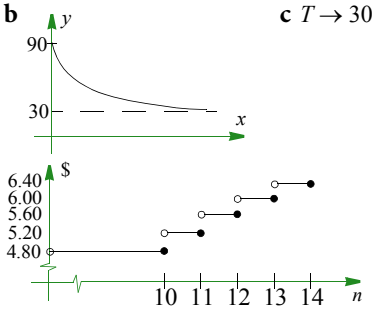
4 b



5 a

t min	1	3	7	14	29	59
T °C	60	45	37.5	34	32	31

b



c $T \to 30$ **d** 2

6

7 a $y = \begin{cases} 0.5x + 1 & \text{if } -2 \le x < 0 \\ 1 - 0.5x & \text{if } 0 \le x \le 2 \end{cases}$ **b** $x = \begin{cases} -1 & \text{if } -3 < x \le -1 \\ 0 & \text{if } -1 < x \le 1 \\ 1 & \text{if } 1 < x \le 3 \end{cases}$

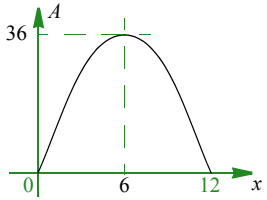
Exercise 9.1.1

1 8
2 4, 0.25
3 8, 18
4 8 and 11 or −8 and −11
5 6, −10
6 2 m
7 51 kmh^{-1}
8 11 and 13 or −11 and −13
9 25 days
10 30
11 a 30 **b** \$50 each
12 6 kmh^{-1}

Exercise 9.1.2

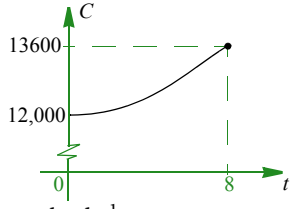
1 a i $100 - 2x$ **ii** $0 < x < 50$ (NB: if $x = 0$ or 50, $A = 0$ and so there is no enclosure)
b i $A = 2x(50 - x), 0 < x < 50$ **ii** 10 m by 80 m or 40 m by 20 m **iii** 1250 m^2
iv 25 m by 50 m

2 a ii $0 < x < 12$ **b i** 20 m^2 **ii** 32 m^2 **iii** 32 m^2
c



d 6 m by 6 m

3 a

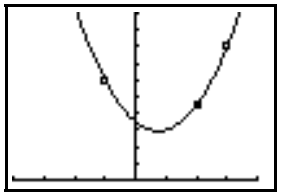


b \$12,900

4 35.83 kmh^{-1}

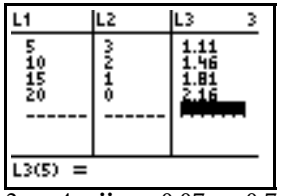
Exercise 9.1.3

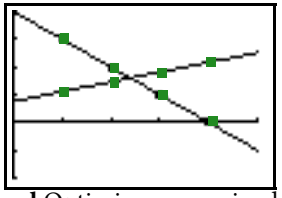
1 a ii $y = 0.4x + 7.2$ **b ii** $y = 6 - 2x$ **c ii** $y = 0.5x + 3.2$
2 Second difference = 0.64
3 b $y = x^2 + 4x + 2$

4 **a** and **c**  **b** $y = 2x^2 - 3x + 7$

5 $y = 2x^2 - x + 3$

Exercise 9.2 Miscellaneous questions

1 **a i** and **ii** 



c i $p = -0.2q + 4$ **ii** $p = 0.07q + 0.76$ **d** Optimium scenario: demand = supply. This occurs when $p = 1.60$, $q = 12$.

2 **a** $y = -\frac{2}{9}x^2 + \frac{11}{9}x + \frac{9}{2}, 0 \le x \le 5.5$ **b** [BE]: $y = -\frac{11}{9}x + \frac{121}{18}, 1 \le x \le 5.5$,
[BO]: $y = 5.5x, 0 \le x \le 1$ **c** 49°36′

9.3 Graded revision questions

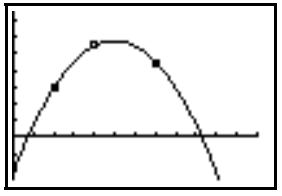
LEVEL 1

1 16

2 6

3 **a** $R(x) = xp = x(40 - 0.0004x), 0 \le x \le 100,000$ **b i** \$960,000
ii 18377 or 81622 (must be integer values) **iii** 1,000,000

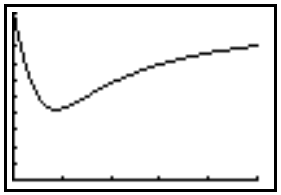
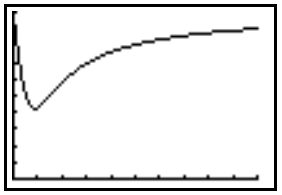
4 **a** $y = \frac{4}{3}(50 - x)$ **b i** $A = \frac{8}{3}x(50 - x)$ **ii** $0 < x < 50$ **c i** $\frac{5000}{3}$ m^2
ii $x = 25, y = \frac{100}{3}$; dimensions 50 m by $\frac{100}{3}$ m

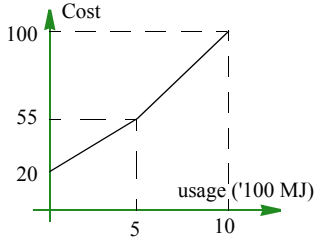
5 **a** Using the coordinates (4, 6), (6, 11) and (9, 9): $y = -0.6333x^2 + 8.8833x - 19.2$

b –40°C at 11 p.m. The model is not valid outside data range. Therefore extrapolation will not necessarily work.

LEVEL 2

1 3 hours

2 35.83 kmh^{-1}

3 **a** 100% **b** $t = 0.229$ (first time) then again at $t = 13.104$
c  
[0, 10] by [0, 1] [0, 20] by [0, 1]
d **i** 42.26% **ii** 1.73 weeks
e As time increases, oxygen level will be 100%

4 **a** 

b \$95.25

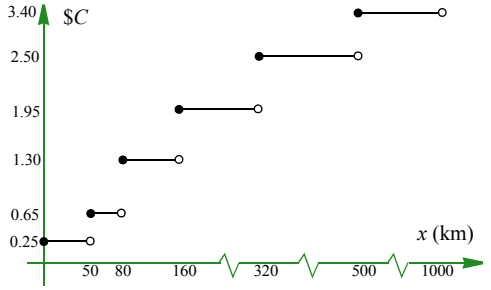
5 Equation of path: $y = -\frac{31}{2400}x^2 + \frac{49}{48}x + 1$. Greatest height: 21.17 m.

LEVEL 3

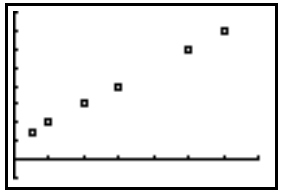
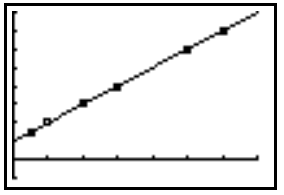
1 9

2 **a** 15 hours **b** 10 hours

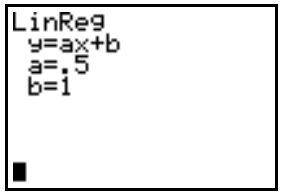
3 B(3254, 1953), C(6146, 3687) units in metres

4 **a** $C = \begin{cases} 0.25, 0 \le x < 50 \\ 0.65, 50 \le x < 80 \\ 1.30, 80 \le x < 160 \\ 1.95, 160 \le x < 320 \\ 2.50, 320 \le x < 500 \\ 3.40, 500 \le x < 1000 \end{cases}$
b 

c \$4.70 **d** \$1.88

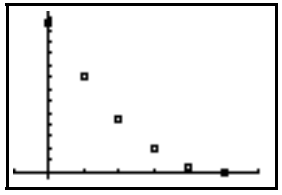
5 **a**  

b linear **c i**

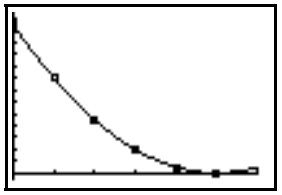


ii $M = 0$, $x = 1$, i.e. 1 m

6 a

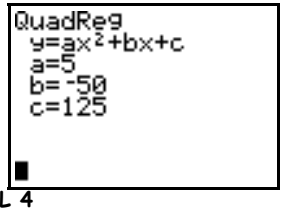


b



c i parabola

d

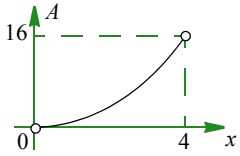


LEVEL 4

1 Chair-one: 20; Chair-two: 24

2 7.5 hours, 10.5 hours

3 a $0 < x < 4$ **b** $A(x) = 3x + 0.25x^2, 0 < x < 4$ **c i**



ii 3.25

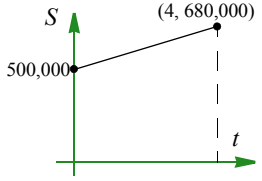
iii $-6 + \sqrt{48}$

4 a i 200 m **ii** 320 m **c** 12 sec **b i** 0.34 sec and 11.66 sec **ii** 11.31 sec **d** 360 m

5 a 0.53 sec (on the way up) and 9.47 sec (on the way down) **b** 10 sec **c** 500 m **d** 12.07 sec **e** 750 m

6 a $P(x) = -2x^2 + x + 3$ **b** $P(x) = (1-k)x^2 + x + k, x \in \mathbb{R}, \neq 1$

7 a i $a = 45{,}000$, $b = 500{,}000$ **ii** $k_1 = 0$, $k_2 = 4$ **b**

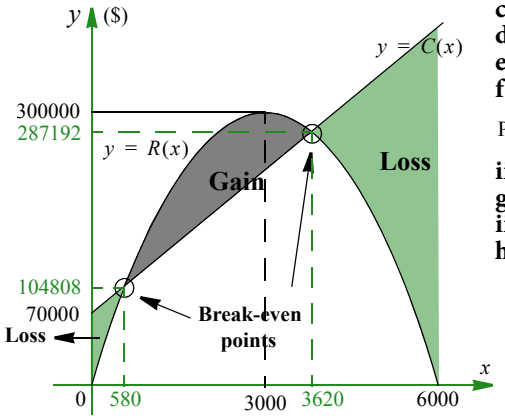


9.4 Topic test

1 a 2 km/day **b** 2.5 km/day

2 a \$72500 **b** No. (Loss of \$20000) **c** 2500

3 a



b i \$70,000 **ii** \$300,000

c Fixed cost (e.g. salary, electricity)

d See graph in **a**

e \$76,667 (to nearest dollar)

f i $P(x) = 140x - \frac{1}{30}x^2 - 70000, 0 \le x \le 6000$

ii \$77,000 **iii** 2100

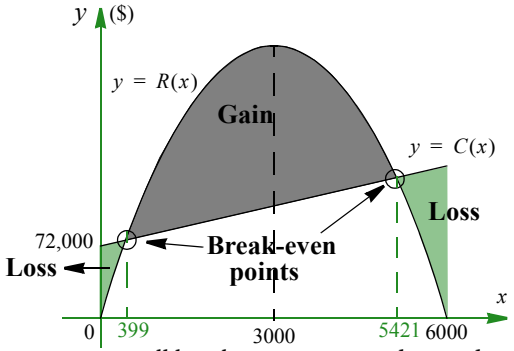
g i $0 \le x \le 580$ or $3620 \le x \le 6000$

ii $581 \le x \le 3619$

h See graph in part **a**

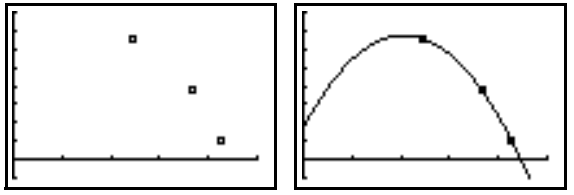
4 b i $P(x) = -\frac{1}{30}x^2 + 194x - 72000$ **ii** $0 \le x \le 6000$

c

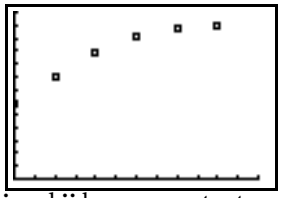


d The company will break-even at 399 radios and 5421 radios. Provided the company sells between 399 and 5421 radios they will make a profit. **e** 2910

5 a



b i Parabolic **ii** $h(x) = -0.04694x^2 + 0.96518x + 1.7896$ **c i** 6.75 m **ii** 22.27 m

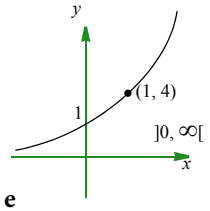
6 a 

b second difference is constant = −50
c $y = -0.25x^2 + 25x + 580$
d \$22,500 per car
e i \$824,750 **ii** \$19,750

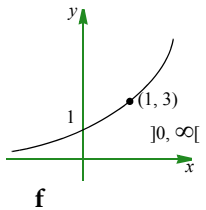
7 a i and **ii** have a constant gradient and so are linear. **iii** use trial and error to verify quadratic relation
b ii $p = 10 - 0.001x$, $C(x) = 2x + 7000$, $R(x) = -0.001x^2 + 10x$
c $P(x) = -0.001x^2 + 8x - 7000$, max profit = $P(4000) = 9000$

Exercise 10.1

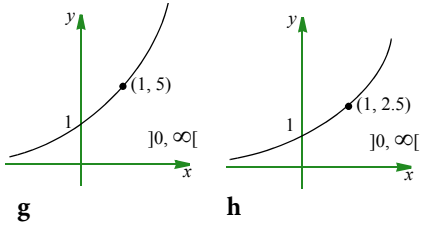
1 a



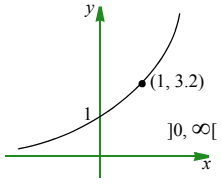
b



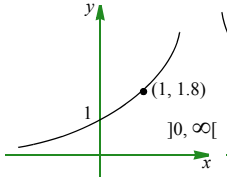
cd



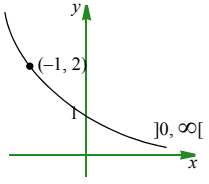
e



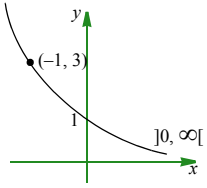
f



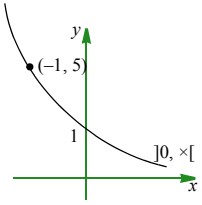
g



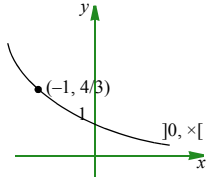
h



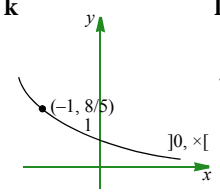
i



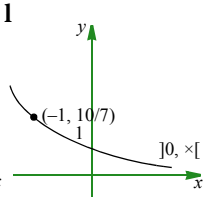
j



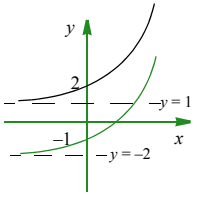
k



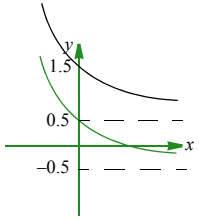
l



2 a

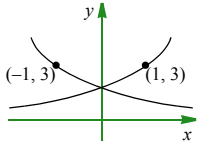


b

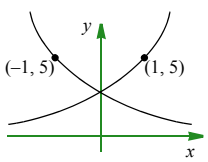


3 '*b*' has a dilation effect on $f(x) = a^x$ (along the *y* axis).

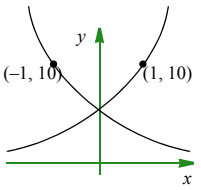
4 a



b

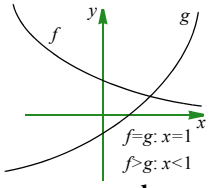


cd

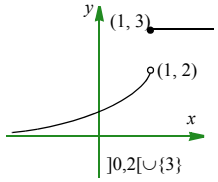


5 a [1, 16] **b** [3, 27] **c** [0.25, 16] **d** [0.5, 4] **e** [0.125, 0.25] **f** [0.1, 10]

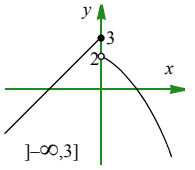
6 a −1, 5 **b**



7 a



b

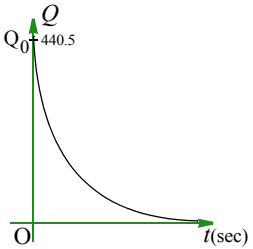


Exercise 10.2

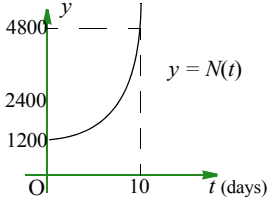
1 a 3 **b** 3 **c** 2 **d** 3.91 **d** 2.93 **e** 1.66 **f** 2.40 **g** 1.76 **h** 4.17 **i** −1/3 **j** −1/3 **k** −1/2 **l** −3.32 **m** 0 **n** −2.32
2 a 1.63 **b** 1.74 **c** 2.32 **d** 1.89

Exercise 10.3

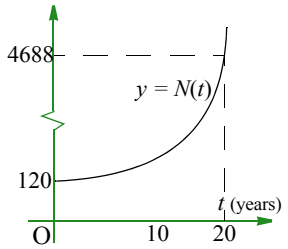
1 a 444.5 **b** 31.63% **c** 6.02 sec **d**



2 a 1200 **b i** 2400 **ii** 4800 **c** 10.3 years (approx.) **d**

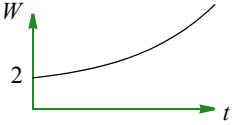


3 **a** 129 **b** 0.1833 **c i** 750 **ii** 900 **d** 11.57 years



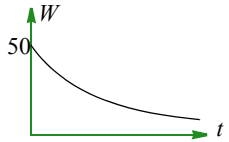
4 **a** 1000 **b i** 1516 **ii** 2000 **c** 10 days

5 **a** 0.0013 **b** 2.061 kg **c** 231.56 years **d**

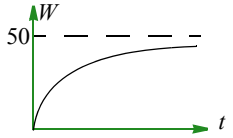


6 **a i** 157 **ii** 165 **iii** 191 **b** 14.2 years **c** 20.1 years **d**

7 **a** 50 **b** 0.0222 **c** 18 kg **d**

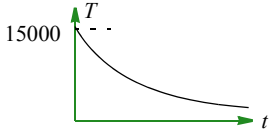


e

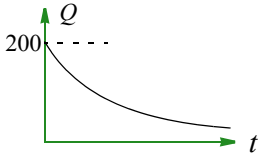


8 **a** 15000°C **b i** 11915°C **ii** 1500°C **c** 3.01 million years

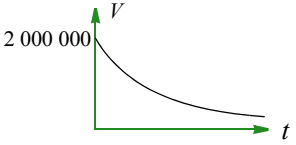
d



9 **a** 0.0151 **b** 12.5 g **c** 20 years **d**



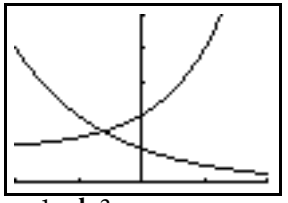
10 **a** \$2 million **b** \$1.589 mill **c** 30.1 years **d**



11 **b** 0.01761 **c** 199 230 **d** 22.6 years

Exercise 10.4 Miscellaneous questions

1 **a**



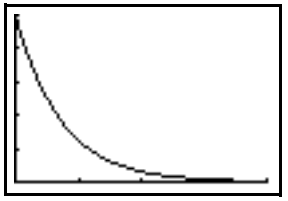
b –0.6

2 **a** $c = 1$ **b** 3

3 **a** 8 16 32 64 128 256 **b i** 45 **ii** 3.6

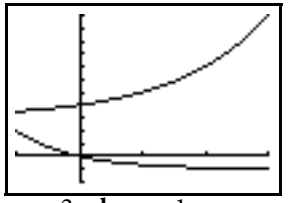
4 **a** 400 **b i** 705 **ii** 1242 **c** 14.2

5 **a**



b 0.5 sec

6

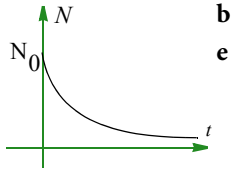


7 **a** $x = 2$ **b** $x = -1$

8 $c = -2$, $k = 0.25$

9 **a** 10 20 40 80 160 320 640 **c i** 56 (or 57) **ii** $t = 5.64$

10 **a**



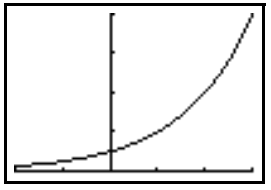
b 9.24% of N_0 **c** $t = 3.50$ (years) **d** 160.43 cm

e 72 days old

10.5 Graded revision questions

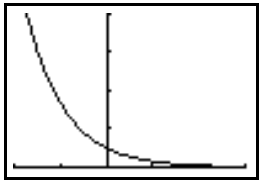
LEVEL 1

1 **a**



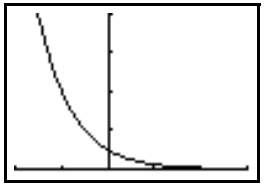
Window : [–2, 3] by [0,8]

b



Window : [–2, 3] by [0,8]

c



Window : [–2, 3] by [0,8]

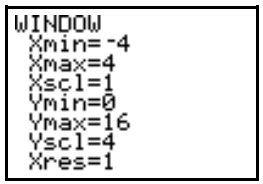
2 **a** 17 **b** 5/3

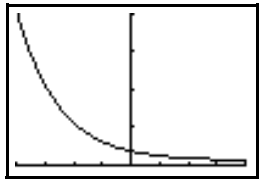
LEVEL 2

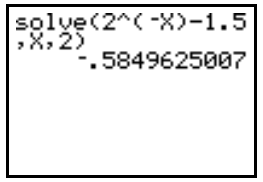
1 **a** 3.6 **b** 2.5 **c** 1.8

LEVEL 3

1





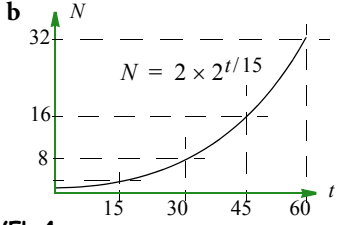


2 $c = 2, k = -2/3$

3 **a**

t seconds	0	15	30	45	60
N	2	4	8	16	32

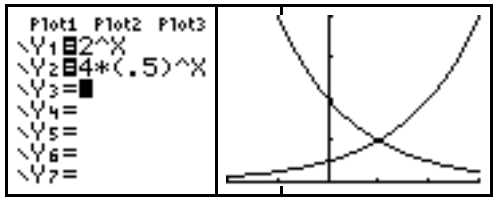
b

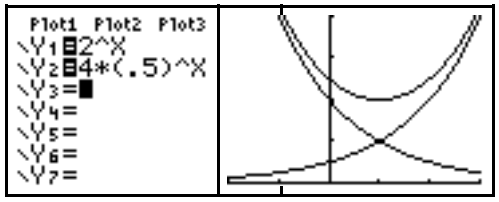


c 2097152

LEVEL 4

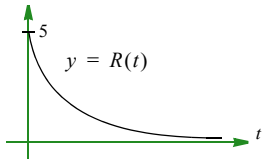
1 **a**



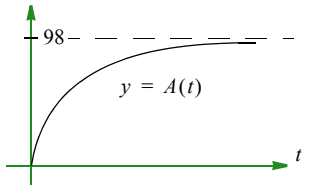


b 0, 2

2 **a** 5 **b i**



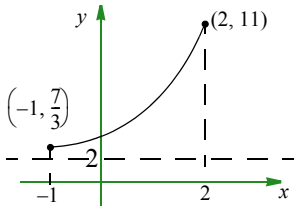
ii



c 49 mg

10.6 Topic test

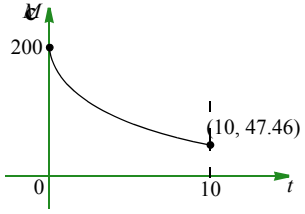
1



2 $a = 8, k = 4, b = 2$

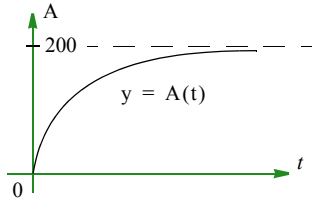
3 **a** $x = 3.170$ (to 3 d.p.) **b** $x = -0.708$ (to 3 d.p.)

0.866 approx **c** 64

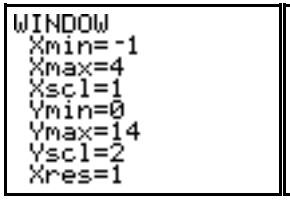


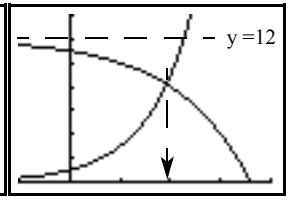
d $A = 200 - M = 200 - 200(0.75)^{t/2}$

$= 200\left[1 - \left(\frac{3}{4}\right)^{t/2}\right]$



5 From the graphs below, we have that $x \sim 1.9$





Exercise 11.1

1 **a** $\sqrt{41}$ **b** $\sqrt{72}$ **c** $\sqrt{225} = 15$ **d** $\sqrt{250}$ **e** $\sqrt{85}$ **f** $\sqrt{173}$ **g** $\sqrt{41}$ **h** $\sqrt{89}$

2 **i** $\sqrt{226}$ **j** $\sqrt{106}$ **k** $\sqrt{205}$ **l** $\sqrt{180}$ **m** $\sqrt{400} = 20$ **n** $\sqrt{145}$ **o** 4

Exercise 11.2

1 **a** 2 **b** 3 **c** $\frac{5}{3}$ **d** 8 **e** 0 **f** 2 **g** −2 **h** undefined

2 **a** $y = 2x - 1$ **b** $y = 3x + 9$ **c** $y = -x - 1$

3 **a** $-\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{3}{2}$ **d** $-\frac{4}{5}$

4 $y = 2x$

5 $y = -x + 1$

6 $2y - x - 2 = 0$

7 ± 12

8 2

9 Parallelogram

10 **a** $y = \frac{5}{2}x$ **b** $y = -\frac{3}{2}x + 3$ **c** $y = \frac{5}{6}x - \frac{1}{2}$ **d** $y = -2x + 1$

Exercise 11.3

11 (0, 1), (2, 5) and (6, 3)

12 **a** $y = 3x + c$ **b** $y = 3x - 8$ **c** (3, 1) **d** $\sqrt{40}$

13 (36.67, 89.5) to (110, 265.5)

Exercise 11.4

1 **a** 1.46 **b** −0.65, 1.25 **c** −1.65, 1.56, 9.92 **d** −1.66 **e** −1.39, −0.14, 1.83, 5.69 **f** 0.59 **g** 0, 0.15, 2.31, 9.35 **h** −0.75, 0.75

2 **b** −0.33, 1.54

3 **b** $m = 1, c = 2$ **c** −2, 1

4 −0.79, 3.79

5 −0.85, 0.53

6 1.70, 0.89, −1.70, −0.89

Exercise 11.5 Miscellaneous questions

1 **a** −3 **b** (2, 0), (6, 0) **c** $\frac{1}{3}$ **d** $3y - x - 3 = 0$

2 $\frac{512}{3}$ cm^3

3 **a** −2 **b** $y = -2x + 8$ **c** $a = 4, b = 3$

4 $7y - 6x - 10 = 0$, $6y + 7x - 24 = 0$

5 **a** $y = 4x - 9$ **b** $y = -x + 1$ **c** $2y - x + 4 = 0$ **d** $2y - x + 4 = 0$

6 **a** $y + 2x - 5 = 0$ **b** $2y - x + 5 = 0$

7 $\left(\frac{7}{6}, \frac{5}{6}\right)$

8 $r = 6$, Area = 118.90 cm^2

9 $b = -1.5, a = -2.5$

10 $y = 2x + 4$

11 $3x - 2y + 2 = 0$

12 (1, 4)

13 a $\sqrt{89}$ **b** 9.43

14 $y = 2x - 2.5$

15 $\frac{1}{6}V$ cm^3

16 1:2

17 $\frac{13}{4}$ m^3

11.6 Graded revision questions

LEVEL 1

1 **a** $m = 3, c = -1$ **b** $m = -1, c = -1$ **c** $m = 0.5, c = 2$

2 **a i** 1 **ii** $\sqrt{109}$ **iii** $2\sqrt{13}$ **b i** 0 **ii** $\frac{10}{3}$ **iii** 1.5 **c i** (2.5, 4) **ii** (3.5, 3) **iii** (0, 0)

3 **a** 2.52 cm^2 **b** $4\pi + 20$ cm^2

LEVEL 2

1 **a** −0.5 **b** 3 **c** −2.5

2 $y = 3x - 3$

3 $2y - 3x - 1 = 0$

4 $2y - x - 3 = 0$

5 **a** $y = -\frac{3}{2}x + 3$ **b** $y = \frac{4}{3}x + \frac{7}{3}$ **c** $y = ax - a^2$

6 $120 + 12\sqrt{13.75}$ cm$^2 \sim 164.50$ cm^2

LEVEL 3

1 **a i** $k = 3$ **ii** $k = -\frac{1}{3}$ **b i** $k = -\frac{2}{3}$ **ii** $k = 6$

2 $\frac{a}{b} = \frac{17}{6}$

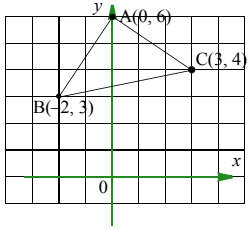
3 $A \equiv (4, 0)$

4 $a = -30, b = -20$

LEVEL 4

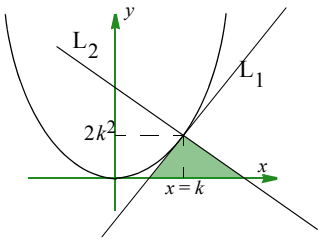
1 $m = \frac{8}{3}$

2 **a**



c $D \equiv (1, 1)$

3 **a**



b $y = 2x^2 \therefore y' = 4x$. So, at $x = k, y' = 4k$, i.e. gradient = $4k$

4 At $(k, 2k^2), m = 4k \therefore (y - 2k^2) = 4k(x - k) \Rightarrow y = 4kx - 2k^2$

5 **c** $4y + x = 16 \Leftrightarrow y = -\frac{1}{4}x + 4 \therefore -\frac{1}{4} \times 4k = -1 \Rightarrow k = 1$

6 **d** For $L_1: y = 0 \Rightarrow 0 = 4kx - 2k^2 \Leftrightarrow x = \frac{2k^2}{4k} = \frac{1}{2}k$

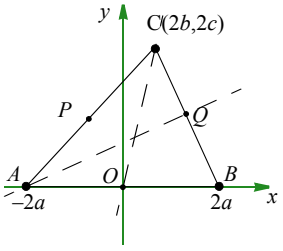
7 For $L_2: y = 0 \Rightarrow 4 \times 0 + x = 16 \Leftrightarrow x = 16$

8 $\therefore \text{Area} = \frac{1}{2} \times \left(16 - \frac{1}{2}\right) \times 2(1)^2 = \frac{1}{2} \times \frac{31}{2} \times 2 = \frac{31}{2}$ sq. units

9 $(a, 0)$

10 **a** $a = 2, -3$ **b** $a = -0.6$

11 **a**

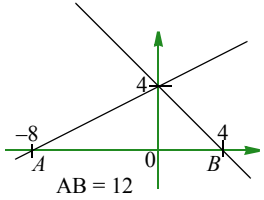


b midpoints: $O \equiv (0, 0)$,

$P \equiv \left(\frac{2b + (-2a)}{2}, \frac{2c + 0}{2}\right) \equiv (b - a, c)$

$Q \equiv \left(\frac{2a + 2b}{2}, \frac{2c + 0}{2}\right) \equiv (a + b, c)$

12 **b**



13 midpoints: $O \equiv (0, 0)$,

$P \equiv \left(\frac{2b + (-2a)}{2}, \frac{2c + 0}{2}\right) \equiv (b - a, c)$

$Q \equiv \left(\frac{2a + 2b}{2}, \frac{2c + 0}{2}\right) \equiv (a + b, c)$

11.7 Topic test

1 **a** $2\sqrt{5}$ **b** –2 **c** (4, 3)

2 $x = -4.5, y = -6$

3 **a** If $pq = 3$ **b** $p = -3q$

4 **a i** $y = -\frac{1}{3}(x - 1)$ **ii** $y = 3(x + 3)$ **b** $X \equiv (-2.6, 1.2)$ **c** 4.8 sq. units

5 **a** $B \equiv (a + b, 0 + c) \equiv (a + b, c)$ **b** For $\overline{AC}$: $\left(\frac{1}{2}(a + b), \frac{1}{2}c\right)$ $\overline{OB}$: $\left(\frac{1}{2}(a + b), \frac{1}{2}c\right)$

c They bisect **d** $B \equiv (b + \sqrt{b^2 + c^2}, c)$

e $m_{OB} = \frac{c}{b + \sqrt{b^2 + c^2}}, m_{AC} = \frac{-c}{\sqrt{b^2 + c^2} - b}$ **f** $m_{OB} \times m_{AC} = -1$. Therefore, diagonals are perpendicular.

Exercise 12.1.1

1 **a ii** 4 **iii** $t_n = 4n - 2$ **b ii** –3 **iii** $t_n = -3n + 23$ **c ii** –5 **iii** $t_n = -5n + 6$
d ii 0.5 **iii** $t_n = 0.5n$ **e ii** 2 **iii** $t_n = y + 2n - 1$ **f ii** –2 **iii** $t_n = x - 2n + 4$

2 –28
3 9, 17
4 –43
5 7
6 7
7 –5
8 0
9 **a** 41 **b** 31st
10 $2, \sqrt{3}$
11 **a i** 2 **ii** –3 **b i** 4 **ii** 11
12 $x - 8y$
13 $t_n = 5 + \frac{10}{3}(n - 1)$
14 **a** –1 **b** 0

Exercise 12.1.2

1 **a** 145 **b** 300 **c** –170
2 **a** –18 **b** 690 **c** 70.4
3 **a** –105 **b** 507 **c** 224
4 **a** 126 **b** 3900 **c** 14th week
5 855
6 **a** 420 **b** –210

Exercise 12.1.3

1 123
2 –3, –0.5, 2, 4.5, 7, 9.5, 12
3 3.25
4 $a = 3$ $d = -0.05$
5 10 000
6 330
7 –20
8 328

9 \$725, 37wks
10 **a** \$55 **b** 2750
11 **a i** 8 m **ii** 40 m **b** 84 m **c** Dist = $2n^2 - 2n = 2n(n-1)$ **d** 8
e 26 players, 1300 m
12 **a** 5050 **b** 10200 **c** 4233
13 **a** 145 **b** 390 **c** –1845
14 **b** $3n - 2$

Exercise 12.2.1

1 **a** $r = 2, u_5 = 48, u_n = 3 \times 2^{n-1}$

b $r = \frac{1}{3}, u_5 = \frac{1}{27}, u_n = 3 \times \left(\frac{1}{3}\right)^{n-1}$ **c** $r = \frac{1}{5}, u_5 = \frac{2}{625}, u_n = 2 \times \left(\frac{1}{5}\right)^{n-1}$

d $r = -4, u_5 = -256, u_n = -1 \times (-4)^{n-1}$ **e** $r = \frac{1}{b}, u_5 = \frac{a}{b^3}, u_n = ab \times \left(\frac{1}{b}\right)^{n-1}$

f $r = \frac{b}{a}, u_5 = \frac{b^4}{a^2}, u_n = a^2 \times \left(\frac{b}{a}\right)^{n-1}$

2 **a** ± 12 **b** $\frac{\pm\sqrt{5}}{2}$

3 **a** ± 96 **b** 15th

4 **a** $u_n = 10 \times \left(\frac{5}{6}\right)^{n-1}$ **b** $\frac{15625}{3888} \approx 4.02$ **c** $n = 5$ (4 times)

5 $-2, \frac{4}{3}$

6 **a i** \$4096 **ii** \$2097.15 **b** 6.2 years

7 $u_n = \frac{1000}{169} \times \left(\frac{12}{5}\right)^{n-1}, \frac{1990656}{4225} \cong 471.16$

8 2.5, 5, 10 or 10, 5, 2.5
9 53 757
10 108 952
11 **a** \$56 156 **b** \$299 284

Exercise 12.2.2

1 **a** 3 **b** $\frac{1}{3}$ **c** –1 **d** $-\frac{1}{3}$ **e** 1.25 **f** $-\frac{2}{3}$

2 **a** 216 513 **b** 1.6384×10^{-10} **c** $\frac{256}{729}$ **d** $\frac{729}{2401}$ **e** $-\frac{81}{1024}$

3 **a** 11; 354292 **b** 7; 473 **c** 8; 90.90909 **d** 8; 172.778 **e** 5; 2.256
f 13; 111.1111111111

4 **a** $\frac{127}{128}$ **b** $\frac{63}{8}$ **c** $\frac{130}{81}$ **d** 60 **e** $\frac{63}{64}$

5 4; 118096
6 \$2109.50
7 9.28 cm
8 **a** $V_n = V_0 \times 0.7^n$ **b** 7
9 54
10 53.5 g; 50 weeks
11 7
12 9
13 –0.5, –0.7997
14 $r = 5, 1.8 \times 10^{10}$
15 \$8407.35
16 1.84×10^{19} ~ 200 billion tonnes.

Exercise 12.2.3

1 Term 9 A.P. = 180, G.P. = 256. Sum to 11 terms A.P. = 1650, G.P. = 2047
2 18
3 12
4 12, 7
5 8 weeks (Ken \$220 and Bo-Youn \$255)
6 **a** week 8 **b** week 12
7 **a** 1.618 **b** 121379 (~121400, depends on rounding errors)

Exercise 12.2.4

1 **a** $\frac{81}{2}$ **b** $\frac{10}{13}$ **c** 5000 **d** $\frac{30}{11}$

2 $23\frac{23}{99}$

3 6667 fish. [NB: $t_{43} < 1$. If we use $n = 43$ then ans is 6660 fish; 20 000 fish. Overfishing means that fewer fish are caught in the long run.

4 27
5 48, 12, 3 or 16, 12, 9
6 **a** $\frac{11}{30}$ **b** $\frac{37}{99}$ **c** $\frac{191}{90}$
7 128 cm
8 $\frac{121}{9}$

9 $2 + \frac{4}{3}\sqrt{3}$

10 3, –0.2
11 $\frac{2560}{93}$

12 $\frac{10}{3}$

13 $\frac{43}{18}, \frac{458}{99}, \frac{413}{990}$

14 9900
15 3275

16 3
17 $t_n = 6n - 14$
18 6
19 $-\frac{1}{6}$
20 **a** 12 **b** 26
21 9 cm, 12 cm
22 ±2
23 (5, 5, 5) [trivial case], (5, –10, 20)
24 **a** 2, 7 **b** 2, 5, 8 **c** $3n - 1$
25 **a** 5 **b** 2 m

Exercise 12.3

1 \$2773.08
2 \$4377.63
3 \$1781.94
4 \$12216
5 \$35816.95
6 \$40349.37
7 \$64006.80
8 \$276971.93, \$281325.41
9 \$63762.25
10 \$98.62, \$9467.14, interest \$4467.52. Flat interest = \$6000
11 \$134.41, \$3790.44, 0.602% /month (or 7.22% p.a)

Exercise 12.4 Miscellaneous questions

1 **a i** 5 **ii** 3 **b** 5, 8, 11, 14, 17 **c** 62 **d** 100
2 29
3 **b i** 10, 22, 34, 46 **ii** 238 **c** $d = 12$, $a = 10$, sum = 640
4 **a** 3 **b** 98415 **c** 147620
5 **a** \$3027 **b** \$5526.70
6 **a** 0.85 **b** \$6264.10 **c i** $u_n = 12000 \times (0.85)^n$ **ii** 8.53 years
7 **a** 5.78 **b** 13 **c** 51.34
8 288
9 –96
10 7 terms
11 1000 numbers
12 **a** $1250 \times 1.12^5 \approx \$2203$ **b** 19 years
13 **a** $A_n = 12000 + 200n$; $B_n = 8000 \times 1.05n$ **b** $n = 13$

12.5 Graded revision questions

LEVEL 1
1 **a** A.P. **b** A.P. **c** Neither **d** G.P.
2 **a** 13 **b** –19 **c** 6 **d** 16
3 **a** 8 **b** 48
4 **a** –40 **b** 1.875 **d** 49152
5 **a** –285 **b** 15 **c** 505 **d** 32769

LEVEL 2
1 45
2 7
3 4782968
4 22 years
5 $b = \frac{a+c}{2}$
6 \$360,000
7 $x = \pm 84$
8 $\frac{121}{9}$
9 9

LEVEL 3
1 **a** 9 **b** 93 **c** 48
2 **a** 313 **b** 1738 **c** 2 **d** 6141
3 28
4 $b = \frac{1}{3}(d + 2a)$

LEVEL 4
1 21
2 21 months
3 15%
4 **a** 4950 **b** 24th day **c** 130 150
5 **c** \$5692.84

12.6 Topic test

1 28
2 425
3 **a** \$26,226.75 **b**

Years since purchase (t)	1	2	3	4
Value of car (\$$V$)	28050	26226.75	24522.01	22928.08

c $V_t = 30000 \times (0.935)^t$, $t = 0, 1, 2,$
4 **a** 10 **b** $S_n = \frac{n}{2}[2(1) + (n-1)1] = \frac{n^2}{2} + \frac{n}{2}$ **c** 10 rows
5 **a** 16 326 ratchets **b** 70 399 ratchets **c** $n = 10.006468$ **d** the 26th year

Exercise 13.1

1 **a** hammer is an element of set A **b** axe is an element of set C **c** Tuesday is an element of set B **d** Tuesday is not an element of set A **e** January is an element of set B **f** Sunday is not an element of set C
2 **a** $32 \in C$ **b** $45 \in N$ **c** Green $\notin$ K **d** Mary $\notin$ P **e** Horse $\notin$ M **f** Banana $\in$ H
3 **a i** T **ii** F **b i** T **ii** T
4 **a** $A = \{x | x = 2x, x = 1, 2, 3, 4\}$ **b** $B = \{x | 40 < x^2 < 169,$ where x^2 is odd$\}$ **c** $C = \{x | 0 < x < 12, x \in \}\mathbb{N}$ **d** $D = \{x | 0 < x < 25,$ and x is a prime number$\}$ **e** $E = \{x | x$ are factors of 28$\}$

5 **a** {4, 9, 16, 25, 36} **b** {(1,6), (4,4), (7,2)} **c** { } or ∅
6 **a** T **b** T **c** F
7 **e** is an empty set. Non empty sets: **a** {1, 3}, **b** {2, 4}, **c** {92}, **d** {3, 4}
8 **a** T **b** F **c** T **d** F **e** F **f** F

Exercise 13.2

1 **a** F **b** T **c** T **d** F **e** F **f** T **g** T **h** F **i** F **j** F **k** F
2 **a** $\frac{5}{11}$ **b** $\frac{316}{99}$ **c** $\frac{1763}{330}$ **d** $\frac{8441}{660}$
3 **a** $x = \frac{8}{5} \therefore x \in \mathbb{Q}$ **b** {(0, 5), (1, 4), (2, 3), (3, 2), (4, 1), (5, 0)} **c** {(1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 3), (2, 4), (2, 5), (3, 1), (3, 2), (3, 4), (3, 5), (4, 1), (4, 2), (4, 3), (4, 4), (5, 1), (5, 2), (5, 3), (0, 6), (6, 0)} **d** $\{x \mid 3 < x \leq 6\}$

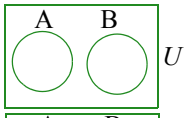
Exercise 13.3

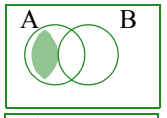
1 **a** T **b** F **c** F **d** T **e** T **f** F **g** T **h** T **i** T
2 **a** 4 **b** 12 **c** 7 **d** 11
3 **a** Finite **b** Infinite **c** Finite **d** Finite **e** Infinite **f** Finite
4 **a** { } **b** {a , e} **c** {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday} **d** {4, 5} **e** {0, 1, 2, 3, 4, 5} **f** {–4, –3, –2, –1, 0, 1, 2, 3, 4, 5}
g Not possible **h** Not possible
5 **a** 0 **b** 2 **c** 7 **d** 2 **e** 6 **f** 10 **g** Infinite **h** Infinite
6 G and H
7 **a** $\{x \mid x \in \mathbb{N}, 3 < x < 18\}$ **b** $\{x \mid x \in \mathbb{R}, x < 12\}$ **c** $\{x \mid x \in \mathbb{Z}, -56 \leq x \leq 45\}$
d $\{x \mid x \in \mathbb{Q}, -5 \leq x \leq 5\}$

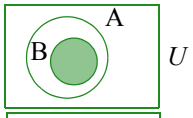
Exercise 13.4

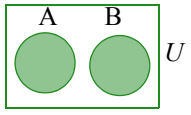
1 **a** T **b** F **c** T **d** T **e** T **f** T **g** F **h** T **i** F
2 **a** T **b** F **c** T **d** T **e** T **f** T **g** F **h** T **i** F
3 { }, {α}, {β}, {μ}, {σ}, {α, β}, {α, μ}, {α, σ}, {β, μ}, {β, σ}, {μ, σ}, {α, β, μ}, {α, β, σ}, {β, μ, σ}, {α, μ, σ}, {α, β, μ, σ}
4 32
5 **a** 31 **b** 15
6 **a** F **b** F **c** T **d** T
7 **a** Yes **b** No **c** Yes **d** Yes
8 **a** {1} {2} {1,2} { } **b** { } **c** {{1}}, {{1}, {1,2}}, {{1}, 3}, {{1,2},3}, {3}, {{1}, {1,2},3}, { }
9 **a** T **b** T **c** F **d** F **e** T **f** F **g** F **h** T
10 6
11 **a** {6, 12, 18} **b** {4, 16} **c** {9} **d** { } **e** {2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18}
f {1, 3, 4, 6, 9, 12, 15, 16, 18} **g** {1, 2, 4, 6, 8, 9, 10, 12, 14, 16, 18}
h {1, 2, 3, 4, 6, 8, 9, 10, 12, 14, 15 16, 18} **i** {4, 9, 16} **j** {3, 4, 6, 9, 12, 15, 16, 18}
k {2, 4, 6, 8, 9, 10, 12, 14, 16, 18}
12 **a** 3 **b** 2 **c** 1 **d** 0 **e** 12 **f** 9 **g** 11 **h** 13
13 **a** {–3, –2} **b** {–3, –2} **c** {–3, –2, –1, 0, 1, 2, 3} **d** {–3, –2}
e {–5, –4, –3, –2, –1, 0, 1, 2, 3} **f** {–3, –2, –1, 0, 1, 2, 3, 4, 5}
g {–5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5} **h** {–5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5}
i {–3, –2, –1, 0, 1, 2, 3} **j** {–3, –2, –1, 0, 1, 2, 3} **k** {–5, –4, –3, –2, –1, 0, 1, 2, 3}
14 **a** 2 **b** 2 **c** 7 **d** 2 **e** 9 **f** 9 **g** 11 **h** 11
15 **a** $\{x \mid x \in \mathbb{R}, -3 \leq x \leq -2\}$ **b** $\{x \mid x \in \mathbb{R}, -3 < x \leq -2\}$
c $\{x \mid x \in \mathbb{R}, -3 < x < 4$ **d** $\{x \mid x \in \mathbb{R}, -3 < x \leq -2\}$ **e** $\{x \mid x \in \mathbb{R}, -5 \leq x < 4\}$
f $\{x \mid x \in \mathbb{R}, -3 \leq x \leq 5\}$ **g** $\{x \mid x \in \mathbb{R}, -5 \leq x \leq 5\}$ **h** $\{x \mid x \in \mathbb{R}, -5 \leq x \leq 5\}$
i $\{x \mid x \in \mathbb{R}, -3 < x < 4\}$ **j** $\{x \mid x \in \mathbb{R}, -3 \leq x < 4\}$ **k** $\{x \mid x \in \mathbb{R}, -5 \leq x < 4\}$
16 **a** { b, d, f, g, j, k, l, n, o, p, q, r, u, v, w, x, y, z } **b** { a, e, i, o, u }
c { c, e, f, g, h, i, k, q, r, s, t, u, v, w, x, y } **d** { o, u } **e** { u } **f** { o, u } **g** { u }
h { b, d, e, f, g, i, j, k, l, n, o, p, q, r, u, v, w, x, y, z }
17 **a** { –10, –9, –8, –7, –6, 3, 4, 5, 6, 7, 8, 9, 10 }
b { –10, –9, –8, –7, –6, –5, –4, –3, –2, 7, 8, 9, 10 }
c { –10, –9, –8, –7, –6, –5, –4, –3, –2, –1, 0, 3, 4, 5, 6,7,8, 9, 10 }
d { –10, –9, –8, –7, –6, 7, 8, 9, 10 **e** { –10, –9, –8, –7, –6, 7, 8, 9, 10 }
f {–10, –9, –8, –7, –6, 7, 8, 9, 10 } **g** { –10, –9, –8, –7, –6, 7, 8, 9, 10 }
h { –10, –9, –8, –7, –6, –5, –4, –3, –2, 3, 4, 5, 6, 7, 8, 9, 10 }
18 **a** $\{x \mid x \notin \mathbb{Z}, x < -5 \text{ or } x > 2\}$ **b** $\{x \mid x \notin \mathbb{Z}, x < -1 \text{ or } x > 6\}$
c $\{x \mid x \notin \mathbb{Z}, x \leq 0 \text{ or } x \geq 3\}$ **d** $\{x \mid x \notin \mathbb{Z}, x < -5 \text{ or } x > 6\}$
e $\{x \mid x \notin \mathbb{Z}, x < -5 \text{ or } x > 6\}$
19 **a** $\{x \mid x \in \mathbb{R}, -10 \leq x \leq -5 \text{ or } -3 < x \leq 10\}$ **b** $\{x \mid x \in \mathbb{R}, -10 \leq x \leq 3\}$
c $\{x \mid x \in \mathbb{R}, -10 \leq x \leq -3, 4 \leq x \leq 10\}$ **d** $\{x \mid x \in \mathbb{R}, -10 \leq x \leq -5 \text{ or } -3 < x \leq 3\}$
e $\{x \mid x \in \mathbb{R}, -10 \leq x \leq -5\}$
20 **a** 28 **b** 19 **c** 25

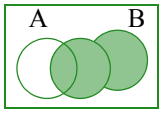
Exercise 13.5

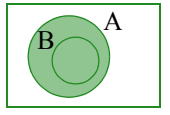
1 **a i** 

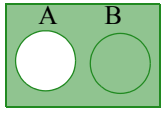
ii 

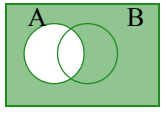
iii 

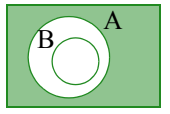
b i 

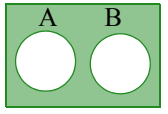
ii 

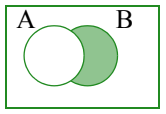
iii 

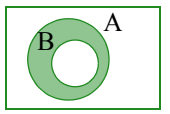
c i 

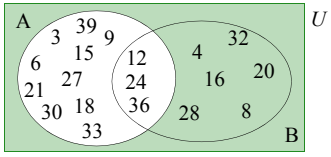
ii 

iii 

d i 

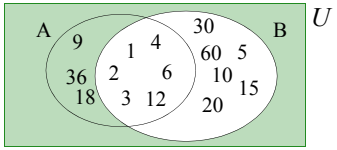
ii 

iii 

2 **a** A = {3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39}
B = {4, 8, 12, 16, 20, 24, 28, 32, 36}
b Multiples of both 3 and 4, less than 40
c **i** 13 **ii** 9 **iii** 3 **iv** 19 **d** 19 = 13 + 9 – 3 **e** 

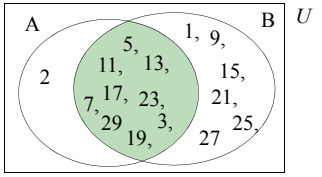
3 **a** A = { 1, 2, 3, 4, 6, 9, 12, 18, 36 }
B = { 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60 }
b Numbers that are both factors of 36 and 60
c i 9 **ii** 12 **iii** 6 **iv** 15 **d** 15 = 12 + 9 – 6

e

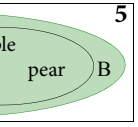


4 **a** A = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29}
B = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29}
b Odd numbers less than 30 that are prime numbers
c i 10 **ii** 15 **iii** 9 **iv** 16 **d** 16 = 10 + 15 – 9

e

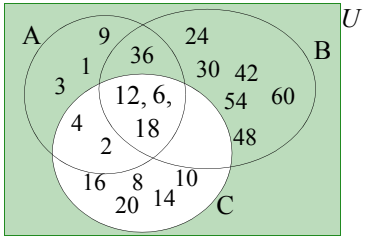


5 **a** **b** The fruits that are common to both sets
c i 5 **ii** 3 **iii** 3 **iv** 5 **d** 5 = 5 + 3 – 3

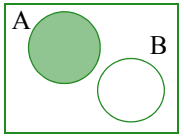


6 **a** A = {1, 2, 3, 4, 6, 9, 12, 18, 36}
B = {6, 12, 18, 24, 30, 36, 42, 48, 54, 60}
C = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}
b Numbers that are factors of 36, multiples of 6 and even.
c i 9 **ii** 10 **iii** 10 **iv** 3 **v** 19

d



7 **a i**



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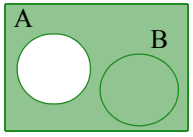

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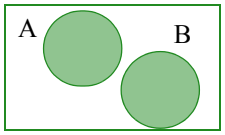
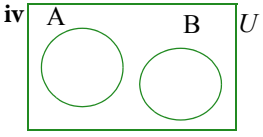
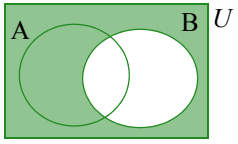


iv



v



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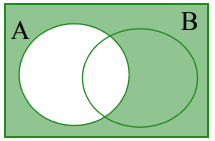
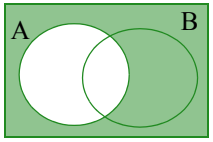
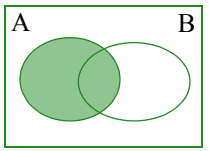


vii



8 **a i**



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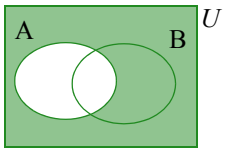
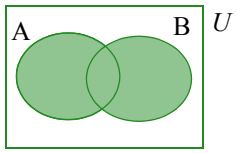
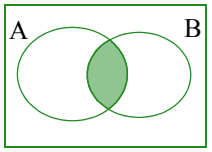


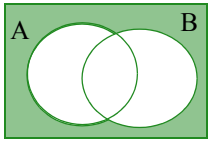
iii



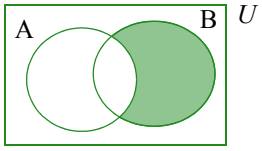
iv



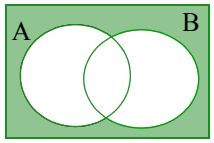
v



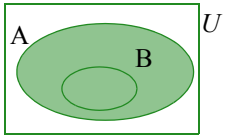
vi



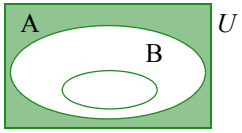
vii



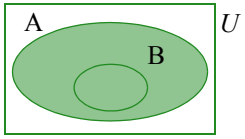
9 **a i**



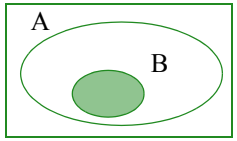
ii



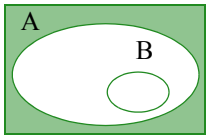
iii



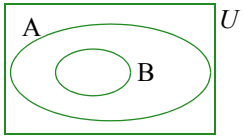
iv



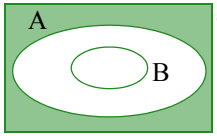
v



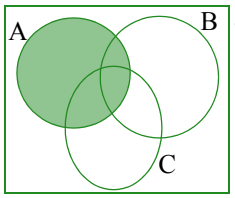
vi



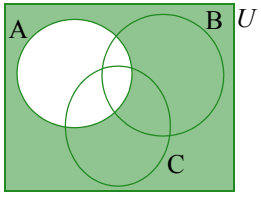
vii



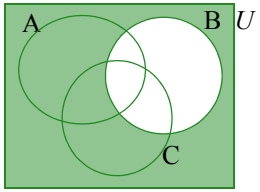
10 **a i**



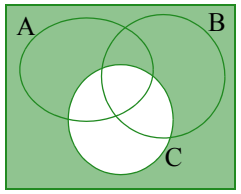
ii

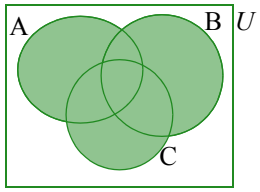


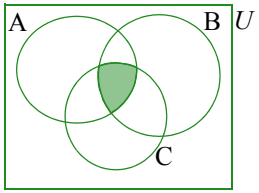
iii

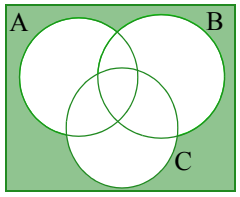


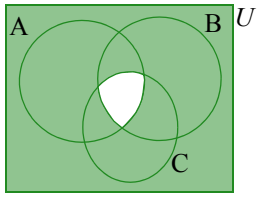
11

iv 

v 

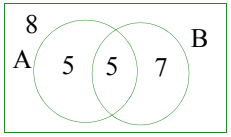
vi 

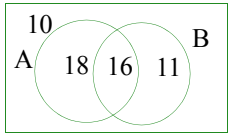
vii 

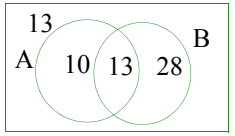
viii 

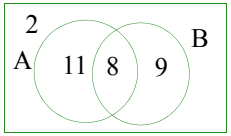
12 **a** A **b** ∅ **c** U **d** A **e** A **f** A **g** A **h** A **i** A ∪ B **j** B ∩ A

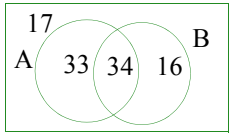
Exercise 13.6.1

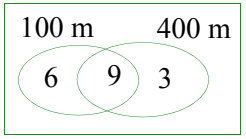
1 **a** 

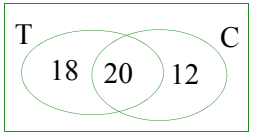
b 

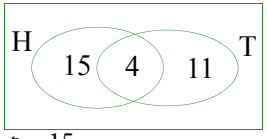
c 

d 

e 

2 

3 

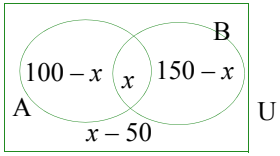
4 

5 $p = 15$

6 Inconsistent information–no Venn diagram possible

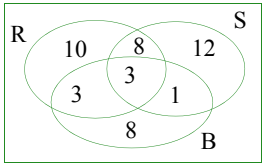
7 **a** 65 **b** 95

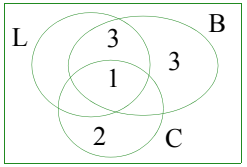
8 240

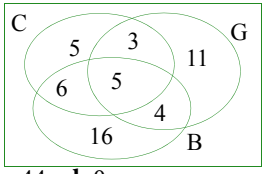
9 

Need: $50 \le x \le 100$

Exercise 13.6.2

1 

2 

3 

4 **a** 44 **b** 0

5 **a** 44 **b** 9

6 18

7 **a** 54 **b** 12

Exercise 13.7 Miscellaneous questions

1 **a** F **b** T **c** F **d** F **e** F **f** T **g** T **h** F **i** T **j** T **k** T **l** T

2 **a** {1, 3, 5} **b** {2, 4} **c** ∅ **d** ∅

3 **a** ∅ **b** {a, e, i, o, u, b, c, d, f, g, h} **c** ∅ **d** {a, e, g, h} **e** {g, h}

4 **a i** {1, 2} **ii** 2 **b i** {6, 8, 10} **ii** 3 **c i** {5, 9, 13, 17, 21} **ii** 5
d i {3, 5, 7, 9, 11, 4, 6, 8, 10, 12, 13, 14, 15, 16} **ii** 14 **e i** {(2,2), (4,4), (6,6)} **ii** 3

5 **a** Finite **b** Infinite **c** Finite **d** Finite **e** Infinite

6 **a** {3, 4, 5 ,6 , . . . } **b** {1, 2, 3, 4,5 ,6 ,7 ,8} **c** {1, 2, 3,4} **d** {1, 2, 3, 4, 5}

7 **a** No **b** Yes

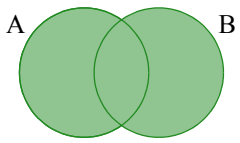
8 **a** {8} **b** {±2} **c** {2}

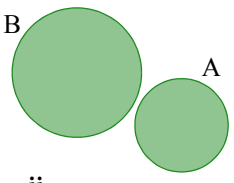
9 **a i** 4 **ii** {0, 4} **iii** 2 **b i** 2 **ii** {1.5, 2, 3} **iii** ∅

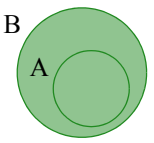
10 **a** {1, 2, 3, 4, 5, 6, 7, 9} **b** {3, 4, 7} **c** {9}

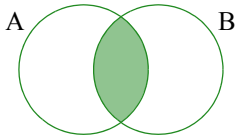
11 A ∩ B = A

12 **ii** and **iv** are correct.

13 **a i**


ii


iii


b i


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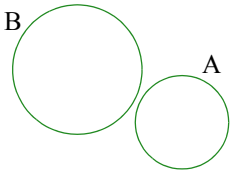

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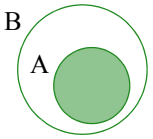
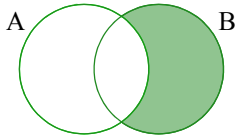


c i


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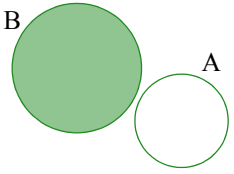

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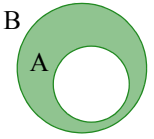
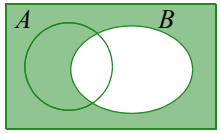
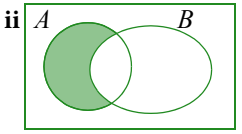


14 **a** 0 **b** 2 **c** 4 **d** 4

15 **a** {0, 1, 2, 3, 4, 5, 6} **b** {7, 8} **c** {0, 1, 5, 6, 7, 8} **d** {0, 1, 2, 3, 4}
e {3, 4, 5, 6, 7, 8} **f** {5, 6}

16 **a** {1, 2, 3, 4, 5} **b** {1} **c** {3, 5} **d** {6, 7, 8, 9} **e** ∅ **f** ∅

17 **a i**


ii


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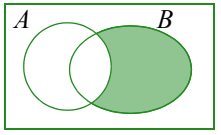
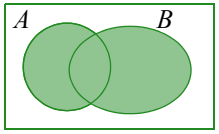
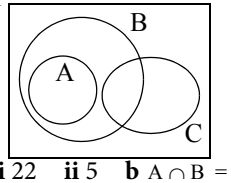


iv


18 **a** F **b** T **c** F

19 **a** 14 **b** 60 **c** 48 **d** 60

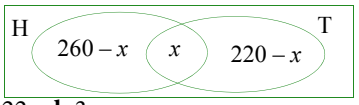
20 **a** 20 **b** 10 **c** 10

21


22 **a i** 22 **ii** 5 **b** $A \cap B = \{5\}$, $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

23 **a** 7 **b** 49

24 **a** $B \cap C = \{x | x \in A$ where x is a perfect square prime number$\}$; $B \cap D = \{x | x \in A$ where x is a prime number whose first digit is greater than its second digit$\}$
b $B \cap D = \{\ \} = \varnothing$, $C \cap D = \{31, 43, 53, 61\}$

25 **a**


b 80

26 **a** 22 **b** 3

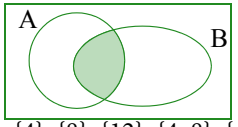
27 **a** 15 **b** 14 **c** 8

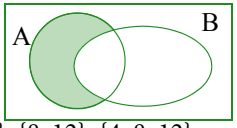
13.8 Graded revision questions

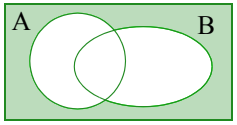
LEVEL 1

1 {3, 4, 5, 6, 7}

2 4

3 **a**


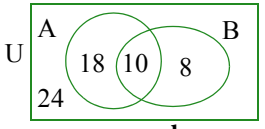
b


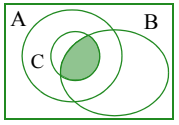
c


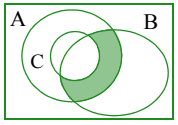
4 { }, {4}, {8}, {12}, {4, 8}, {4, 12}, {8, 12}, {4, 8, 12}

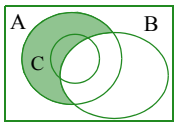
5 16

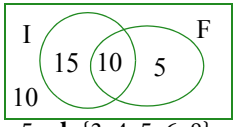
LEVEL 2

6


7 **a**


b


c


8


9 **a** 5 **b** {3, 4, 5, 6, 8}

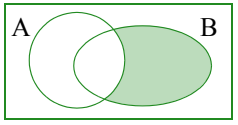
LEVEL 3

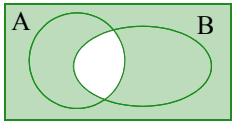
1 **a** 1 or 2 **b** 0 or 1

LEVEL 4

1 $A' \cup B' \cup (A \cap B \cap C') = (A \cap B \cap C)'$

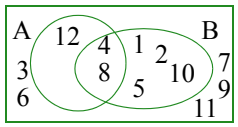
13.9 Topic test

1 **a**


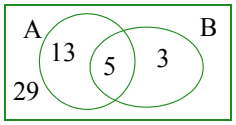
b


2 **a i** {4, 8, 12} **ii** {1, 2, 4, 5, 8, 10}

b



3

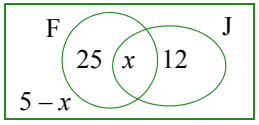


$n(A \cap B') = 13$

4 **a**

	The Fanom is their favourite	The Fanom is not their favourite	Total
Justecia is their favourite	x	12	$12 + x$
	25	$5 - x$	$30 - x$
Total	$25 + x$	$15 - x$	42

5 **b**



c 12

6 **a** 12 **b** 17 **c** 3

Exercise 14.1.1

1 a, b, d, f, h, i, k, l

Exercise 14.1.2

1 **a** Brutus is not sick. **b** The cup is not empty. **c** Monday is not a holiday.
d There are not twelve months in a year. **e** 4 is not less than 6.
f Ismael does not study Geography. **g** Yoshi does not play football.
h All roses are not red. **i** March does not have 31 days.
j Not all quadrilaterals are rectangles. **k** Carlos does not live in Japan

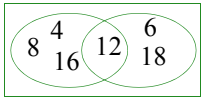
Exercise 14.1.3

1 **a** All trees are green and all mammals are warm blooded. **b** Josh and Anne study French. **c** Abdul plays both football and squash. **d** The Moon orbits Earth and Earth orbits the Sun. **e** It is raining and today is Saturday.

2 **a** p: Jennifer studies Physics. q: Jennifer studies Chemistry. **b** p: All mammals are warm blooded. q: All roses are red. **c** p: Ruth is 16 years old. q: Janet is 17 years old. **d** p: Today is Sunday. q: It is fine. **e** p: Ronnie plays football. q: Renee plays tennis.

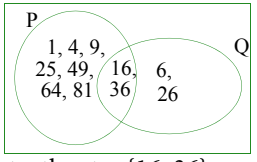
3 **a i** P = {4, 8, 12, 16}, Q = {6, 12, 18} **ii** $p \wedge q$ truth set = {12}

iii U



b i P = { 1, 4, 9, 16, 25, 36, 49, 64, 81 } Q = { 6, 16, 26, 36 }

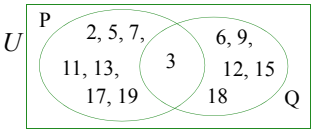
ii U



iii $p \wedge q$ truth set = {16, 36}

c i P = { 2, 3, 5, 7, 11, 13, 17, 19 } Q = { 3, 6, 9, 12, 15, 18 }

ii U

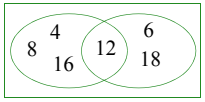


iii $p \wedge q$ truth set = { 3 }

4 **a** All trees are green or all mammals are warm blooded.
b Josh or Anne study French. **b** Abdul plays football or squash. **d** The Moon orbits Earth or Earth orbits the Sun. **e** It is raining or today is Saturday.

5 **a** p: Josh will buy a TV. q: Josh will buy a stereo.
b p: Yoshi plays tennis. q: Mohammed plays squash.
c p: The Moon orbits Earth. q: The Moon orbits the Sun.
d p: Emus are birds. q: Goats are horses.
e p: Ruth likes watching the theatre. q: Ruth likes watching sports.

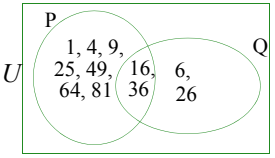
6 **a i** P = { 4, 8, 12, 16 } Q = { 6, 12, 18 } **ii** U



iii $p \vee q$ truth set = { 4, 6, 8, 12, 16, 18 }

b i P = {1, 4, 9, 16 25, 36, 49, 64, 81} Q = { 6, 16, 26, 36}

U

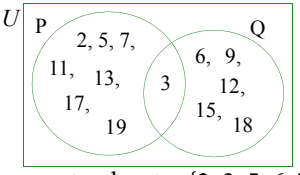


ii

iii $p \vee q$ truth set = { 1, 4, 6, 9, 16, 25, 26, 36, 49, 64, 81}

c i P ={2, 3, 5, 7, 11, 13, 17, 19} Q = { 3, 6, 9, 12, 15, 18 }

ii U



iii $p \vee q$ truth set = {2, 3, 5, 6, 7, 9, 11, 12, 13, 15, 17, 18, 19}

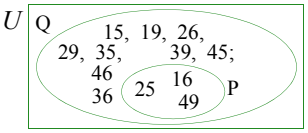
Exercise 14.1.4

1 **a** If x is a multiple of 9 then x is divisible by 3. **b** If Yoshi is sick then Yoshi is not at work. **c** If Paris is in France then the sky is blue. **d** If this is the right time for an argument then it is the right place for an argument. **e** If John works hard then John earns money.

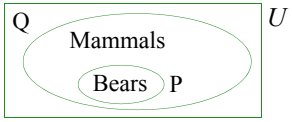
2 **a** p: A number is divisible by 10. q: A number ends in zero.
b p: I will earn more money. q: I work hard.
c p: The flowers will grow. q: There is enough rain.
d p: A number is divisible by 4. q: A number is even.
e p: I think. q: I am.

3 **a** **i** P = { 16, 25, 49 } Q = { 15, 16, 19, 25, 26, 29, 35, 36, 39, 45, 46, 49 }

ii



b



Exercise 14.1.5

1 **a** Converse: If a number ends in zero then it is divisible by 10.
Inverse: If a number is not divisible by 10 then it does not end in zero.
Contrapositive: If a number does not end in zero then it is not divisible by 10.
b Converse: I work hard only if I will earn more money.
Inverse: If I will not earn more money then I will not work hard.
Contrapositive: If I do not work hard then I will not earn more money.
c Converse: There is enough rain only if the flowers will grow.
Inverse: The flowers will not grow only if there is not enough rain.
Contrapositive: If there is not enough rain then the flowers will not grow.
d Converse: If a number is even then it is divisible by four.
Inverse: If a number is not divisible by four then it is not even.
Contrapositive: If a number is not even then it is not divisible by four.
e Converse: If I am then I think.
Inverse: If I do not think then I am not.
Contrapositive: If I am not then I do not think.

Exercise 14.1.6

1 **a** I will paint the house if and only if it is fine. **b** We will go to Disneyland if and only if there are exciting rides. **c** I will take my umbrella if and only if it is raining. **d** John will hit a home run if the pitcher is useless.

2 **a**

p	q	$\neg p$	$\neg p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

b

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

c

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

d

p	q	$\neg p$	q	$\neg p \Rightarrow q$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	F	F

e

p	q	$\neg q$	$p \vee \neg q$	q	$(p \vee \neg q) \Rightarrow$ q
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	T	T
F	F	T	T	F	F

f

p	q	$\neg p$	$\neg p \Leftrightarrow q$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	T	F

g

p	q	$\neg q$	$p \vee \neg q$	$p \wedge \neg q$	$(p \vee \neg q) \Rightarrow (p \wedge \neg q)$
T	T	F	T	F	F
T	F	T	T	T	T
F	T	F	F	F	T
F	F	T	T	F	F

h

p	q	$\neg p$	$\neg p \wedge q$	$q \wedge p$	$(\neg p \wedge q) \vee (q \wedge p)$
T	T	F	F	T	T
T	F	F	F	F	F
F	T	T	T	F	T
F	F	T	F	F	F

i

p	q	$\neg q$	$p \Rightarrow \neg q$	$\neg(p \Rightarrow \neg q)$
T	T	F	F	T
T	F	T	T	F
F	T	F	T	F
F	F	T	T	F

j

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \Rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

k

p	q	$p \wedge q$	$\neg(p \wedge q)$	$q \Leftrightarrow p$	$\neg(q \Leftrightarrow p)$	$\neg(p \wedge q) \vee \neg(q \Leftrightarrow p)$
T	T	T	F	T	F	F
T	F	F	T	F	T	T
F	T	F	T	F	T	T
F	F	F	T	T	F	T

Exercise 14.1.7

1 a

p	q	r	$p \vee q$	r	$(p \vee q) \Rightarrow r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	T	T	T
T	F	F	T	F	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	F	T	T
F	F	F	F	F	T

b

p	q	r	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$(\neg p \vee \neg q) \wedge r$
T	T	T	F	F	F	F
T	T	F	F	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	F
F	T	T	T	F	T	T
F	T	F	T	F	T	F
F	F	T	T	T	T	T
F	F	F	T	T	T	F

c

p	q	r	$\neg p$	$\neg q$	$\neg r$	$\neg p \wedge \neg q \wedge \neg r$
T	T	T	F	F	F	F
T	T	F	F	F	T	F
T	F	T	F	T	F	F
T	F	F	F	T	T	F
F	T	T	T	F	F	F
F	T	F	T	F	T	F
F	F	T	T	T	F	F
F	F	F	T	T	T	T

d

p	q	r	$\neg p$	$q \vee r$	$\neg p \Rightarrow (q \vee r)$
T	T	T	F	T	T
T	T	F	F	T	T
T	F	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	F	F

e

p	q	r	$\neg q$	$p \vee \neg q$	r	$(p \vee \neg q) \Rightarrow r$
T	T	T	F	T	T	T
T	T	F	F	T	F	F
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	T	F	F	F	F	T
F	F	T	T	T	T	T
F	F	F	T	T	F	F

f

p	q	r	$\neg p$	$\neg p \Leftrightarrow q$	$(\neg p \Leftrightarrow q) \vee r$
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	F	T
F	F	F	T	F	F

g

p	q	r	$\neg r$	$p \vee \neg r$	$\neg q$	$r \wedge \neg q$	$(p \vee \neg r) \Rightarrow (r \wedge \neg q)$
T	T	T	F	T	F	F	F
T	T	F	T	T	F	F	F
T	F	T	F	T	T	T	T
T	F	F	T	T	T	F	F
F	T	T	F	F	F	F	T
F	T	F	T	T	F	F	F
F	F	T	F	F	T	T	T
F	F	F	T	T	T	F	F

h

p	q	r	$\neg p$	$\neg p \wedge q$	$q \wedge r$	$(\neg p \wedge q) \vee (q \wedge r)$
T	T	T	F	F	T	T
T	T	F	F	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	T	T	T	T
F	T	F	T	T	F	T
F	F	T	T	F	F	F
F	F	F	T	F	F	F

i

p	q	r	$p \wedge r$	$r \vee q$	$(p \wedge r) \Rightarrow (r \vee q)$
T	T	T	T	T	T
T	T	F	F	T	T
T	F	T	T	T	T
T	F	F	F	F	T
F	T	T	F	T	T
F	T	F	F	T	T
F	F	T	F	T	T
F	F	F	F	F	T

j

p	q	r	$p \wedge q$	$\neg(p \wedge q)$	$q \Leftrightarrow r$	$\neg(q \Leftrightarrow r)$	$\neg(p \wedge q) \vee \neg(q \Leftrightarrow r)$
T	T	T	T	F	T	F	F
T	T	F	T	F	F	T	T
T	F	T	F	T	F	T	T
T	F	F	F	T	T	F	T
F	T	T	F	T	T	F	T
F	T	F	F	T	F	T	T
F	F	T	F	T	F	T	T
F	F	F	F	T	T	F	T

Exercise 14.1.8

1 **a** valid **b** Invalid **c** Invalid

Exercise 14.2 Miscellaneous questions

1 **a** Yes **b** No **c** No **d** No **e** Yes **f** Yes

2 **a** Implication: If Fuzzy is a bear then Fuzzy is cute.
Converse: If Fuzzy is cute then Fuzzy is a bear.
Inverse: If Fuzzy is not a bear then Fuzzy is not cute.
Contrapositive: If Fuzzy is not cute then Fuzzy is not a bear.
b Implication: If Mary lives in Spain then Mary loves fish.
Converse: If Mary loves fish then Mary lives in Spain.
Inverse: If Mary does not live in Spain then Mary does not love fish.
Contrapositive: If Mary does not love fish then Mary does not live in Spain.
c Implication: If it is fine then we will go to the concert.
Converse: We will go to the concert only if it is fine.
Inverse: If it is not fine then we will not go to the concert.
Contrapositive: We will not go to the concert only if it is not fine.
d Implication: If I work hard then I will pay taxes.
Converse: I will pay taxes only if I work hard.
Inverse: If I do not work hard then I will not pay taxes.
Contrapositive: I will not pay taxes only if I do not work hard.
e Implication: If John loves fishing then John lives by the sea.
Converse: If John lives by the sea then John loves fishing.
Inverse: If John does not love fishing then he does not live by the sea.
Contrapositive: If John does not live by the sea then John does not love fishing.

3 **a**

4 row 2 and row 3

5 All

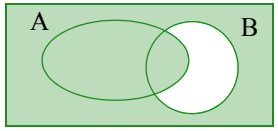
6 Tautology

7 **a** and **b**

8

p	q	$p \wedge q$	$\neg p$	$\neg q$	$(\neg p \wedge \neg q)$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
F	F	F	T	T	T	T
F	T	F	T	F	F	F
T	F	F	F	T	F	F
T	T	T	F	F	F	T

9 **a**



10 **b** $p \vee \neg q$

11 **a** $A \cap B$ **b** $A \cup (B \cup C)$ **c** $A \cap (B \cup C)$

12 $p \vee q$

13 **a** p **b** p **c** p **d** q **e** T (Tautology) **f** p

14 c

15

p	q	$\neg p$	$\neg p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

16 **e**

17 **a** p: Today is Tuesday. $\neg p$ **b** p: x is a even number. q: x is a prime number. $p \vee q$
c p: Mary studies French. q: John studies French. $p \wedge q$
d p: It is raining. q: The concert will be cancelled. $p \Rightarrow q$
e p: Yoshi will go to the concert. q: It is raining. $p \Leftrightarrow \neg q$
f p: Birgit likes ice-cream. q: Birgit likes cake. $p \wedge q$
g p: Jessica will go to the concert. q: Mary goes to the concent.
r: It is raining. $p \Leftrightarrow (q \wedge \neg r)$
h p: It is fine. q: Temperature is between 20°C and 30°C.
r: Paul will play tennis. $(p \wedge q) \Rightarrow r$
i p: I work hard. q: I will pass my exams. $p \Rightarrow q$
j p: Bill wins his race. q: Bill will make the final. $p \Rightarrow q$
k p: Bill wins his race. q: Bill will make the final. $\neg p \Rightarrow \neg q$

18 **a** Invalid **b** Invalid **c** Invalid **d** Valid **e** Invalid

19 **c**

20 **a** No **b** Yes **c** $q \Rightarrow p$

14.3 Graded revision questions

LEVEL 1

1 If x is not a blet, then x is not a flib.

2 "I will not go swimming." is given by $\neg q$. "If it is hot, then I will not go swimming." is the implication. Therefore, p implies the negative of q, i.e. $p \Rightarrow \neg q$.

3 **a** If Physics is difficult, then I don't study hard. **b** If I don't study hard, then Physics is difficult. **c** Physics is difficult if and only if I don't study hard.
d If I do not study hard, then Physics is not difficult.

LEVEL 2

1 **a** $\neg p \Rightarrow q$ is an implication, stating the negative of p implies q. Therefore the converse is $q \Rightarrow \neg p$ **b** "She is not tall." is represented by $\neg x$. "She is not tall and she is beautiful." is the conjunction of $\neg x$ and y, i.e. $\neg x \wedge y$.

2 **e**

LEVEL 3

1 **a**

p	q	$\neg p$	$\neg p \Rightarrow q$	$p \vee q$	$(\neg p \Rightarrow q) \Leftrightarrow (p \vee q)$
T	T	F	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	F	T	F	F	T

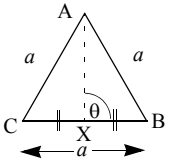
2 **b** Regardless of the truth value of p and q, the result is always true. Therefore, $(\neg p \Rightarrow q) \Leftrightarrow (p \vee q)$ is a tautology.
c If I do not train, I will get into trouble. $p \vee q$ is the disjunction of p and q, that is, "I train or I get into trouble". Therefore, either p and q are both true, or at least one of them is true.

3 **a** $x + xy = x(1 + y) = x(1) = x$ **b** $xy' + x'y + xy = x + y$ (Use a Venn diagram.)

4 Valid

LEVEL 4

1 Consider the contrapositive approach:
$\neg q$ i.e. the triangle is equilateral
$\neg p$ i.e. the line drawn from the vertex of a triangle to the midpoint of the opposite side does intersect this side at a right angle.
Therefore, we must prove that $\neg q \Rightarrow \neg p$.
Consider the ΔABC as shown in the diagram: Given that $AB = AC$ and $BX = CX$.



then, as AX is common we have $\Delta ABX \cong \Delta ACX$ (SSS)
$\therefore \angle AXB = \angle AXC$
$\therefore \angle AXB = 90°$ (straight line)
Hence, $\neg q \Rightarrow \neg p$ which is equivalent to $p \Rightarrow q$ (as required).
Or very simply: $\neg q \Rightarrow \neg p$ so $\neg(\neg q) \Rightarrow \neg(\neg p)$ or $p \Rightarrow q$

2 Use truth table to show that the last column consists of Ts only.

14.4 Topic test

1 p:The movie is over. q: We go home.

2 A: $x \wedge y$ is the conjunction of x and y. It is true only if both x and y are true. Since y is false, then $x \wedge y$ is false.
B: $x \vee y$ is the disjunction of x and y. It is true if either x or y is true. If x is true then $x \vee y$ is true.
C: The statement y is false because $3 \times 4 = 12$ ($\nmid 18$).
D: $x \Rightarrow y$ is the implication, if x is true then y is true. However, x is true but y is false.

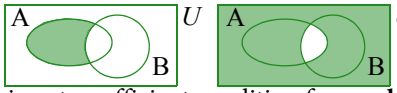
3 **a** If x is prime and odd, then $x > 2$. **b** $(p \wedge r) \Rightarrow q$

4 **a**

p	q	$p \Rightarrow q$	$\neg q$	$p \wedge \neg q$	$\neg(p \wedge \neg q)$	$(p \Rightarrow q) \Leftrightarrow \neg(p \wedge \neg q)$
T	T	T	F	F	T	T
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

5 **b** The statement "If Nora lives in Sydney, then Nora lives in NSW" is represented by $p \Rightarrow q$.
However, the statement "It is false that Nora lives in Sydney and Nora does not live in NSW" is given by $\neg(p \wedge \neg q)$.

6 **a** He does not eat too much. **b** He does not eat too much and he is healthy.
c He is healthy or happy. **d** If he eats too much then he is not happy.
e If he does not eat too much and he is healthy than he is happy.
f He eats too much if and only if he is not happy. **g** He eats too much or he is happy. **h** If he does not eat too much or he is healthy than he is happy.
i If he does not eat too much then he is healthy and happy.
j He is happy if and only if he does not eat too much and he is healthy.

7 **a i** 

b i $A \cap B'$ **ii** $(A \cap B)' \cup (A \cap B)$

8 **a** p is not a sufficient condition for q. **b** q is a sufficient condition for p. **c** $q \Rightarrow p$

Exercise 15.1

1 **a** $\frac{2}{5}$ **b** $\frac{3}{5}$ **c** $\frac{2}{5}$

2 **a** $\frac{2}{7}$ **b** $\frac{5}{7}$

3 **a** $\frac{5}{26}$ **b** $\frac{21}{26}$

4 {HH, HT, TH, TT} **a** $\frac{1}{4}$ **b** $\frac{3}{4}$

5 {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT} **a** $\frac{3}{8}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$

6 **a** $\frac{2}{9}$ **b** $\frac{2}{9}$ **c** $\frac{2}{3}$ **d** $\frac{1}{3}$

7 **a** $\frac{1}{2}$ **b** $\frac{3}{10}$ **c** $\frac{9}{20}$

8 **a** $\frac{11}{36}$ **b** $\frac{1}{18}$ **c** $\frac{1}{6}$ **d** $\frac{5}{36}$

9 {GGG, GGB. GBG, BGG, BBB, BBG, BGB, GBB} **a** $\frac{1}{8}$ **b** $\frac{3}{8}$ **c** $\frac{1}{2}$

10 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{1}{4}$

11 **a** $\frac{3}{8}$ **b** $\frac{1}{4}$ **c** $\frac{3}{8}$ **d** $\frac{3}{4}$

12 **a**{(1, H), (2, H), (3, H), (4,H), (5, H), (6, H), (1, T), (2, T), (3, T), (4, T), (5, T), (6,T)}
b $\frac{1}{4}$

13 **a** $\frac{1}{216}$ **b** $\frac{1}{8}$ **c** $\frac{3}{8}$

Exercise 15.2

1 **a** $\frac{1}{4}$ **b** $\frac{5}{8}$ **c** $\frac{3}{4}$

2 **a** $\frac{1}{13}$ **b** $\frac{1}{2}$ **c** $\frac{1}{26}$ **d** $\frac{7}{13}$

3 $\frac{9}{26}$

4 **a** 1.0 **b** 0.3 **c** 0.5

5 **a** 0.65 **b** 0.70 **c** 0.65

6 **a** 0.95 **b** 0.05 **c** 0.80

7 **a** {TTT, TTH, THT, HTT, HHH, HHT, HTH, THH} **b** $\frac{3}{8}$ **c** $\frac{1}{2}$ **d** $\frac{1}{4}$ **e** $\frac{3}{8}$

8 **a** $\frac{6}{25}$ **b** $\frac{6}{25}$ **c** $\frac{13}{25}$

9 **b** $\frac{3}{4}$ **c** $\frac{1}{2}$ **d** $\frac{1}{6}$ **e** $\frac{7}{12}$

10 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{8}{13}$ **d** $\frac{7}{13}$

11 **a** 0.1399 **b** 0.8797 **c** 0.6

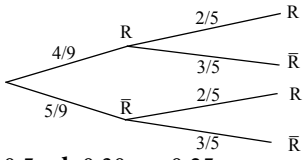
12 **b** $\frac{4}{15}$ **c** $\frac{4}{15}$ **d** $\frac{11}{15}$

Exercise 15.3

1 **a** 0.7 **b** 0.75 **c** 0.50 **d** 0.5

2 **a** 0.5 **b** 0.83 **c** 0.10 **d** 0.90

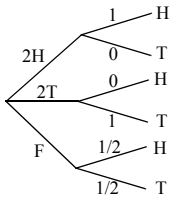
3 **a**



b $\frac{8}{45}$ **c** $\frac{22}{45}$ **d** $\frac{6}{11}$

4 **a** 0.5 **b** 0.30 **c** 0.25

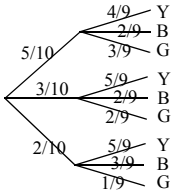
5 **a**



b $\frac{1}{2}$ **c** $\frac{2}{3}$

6 $\frac{1}{3}$

7 **a**



b $\frac{31}{45}$ **c** $\frac{2}{9}$

8 $\frac{2}{3}$

9 **a** 0.88 **b** 0.42 **c** 0.6 **d** 0.28

10 **a** 0.33 **b** 0.49 **c** 0.82 **d** 0.551

11 **a** 0.22 **b** 0.985 **c** 0.8629

12 **a** 0.44 **b** 0.733

13 **a** 0.512 **b** 0.128 **c** 0.8571

14 **a** 0.2625 **b** 0.75 **c** 0.4875 **d** 0.7123

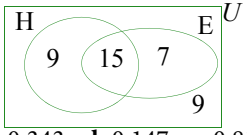
15 **a** 0.027 **b** 0.441 **c** 0.4532

Exercise 15.4 Miscellaneous questions

1 **a** 0.16 **b** 0.70 **c** 0.58

2 **a** 0.5 **b** 0.7 **c** 0.4 **d** 0.2

3 **a**



b **i** 0.375 **ii** 0.225 **iii** 0.4 **iv** 0.225

4 **a** 0.343 **b** 0.147 **c** 0.8125

5 **a** Use of diagram (e.g. Venn), 19 **b** $\frac{1}{7}$ **c** $\frac{19}{37}$ **d** Dependent

6 **a** $\frac{1}{9}$ **b** $\frac{4}{9}$ **c** $\frac{2}{3}$

7 **a**

Number of eggs	2	3	4	5	6	7	8	9	10	11
Frequency	5	1	5	8	3	4	2	6	2	4

b $\frac{3}{20}$ **c** 6.325 eggs per bird **d** $\frac{9}{20}$ **e** 2.787 eggs

f The interval is 3.5376 to 9.11236 or in whole numbers 4 to 9. **g** $\frac{7}{10}$

8 **a** Each spin is physically separate from the others. The result of the first spin will not affect the second.

b **c** 0.36 **d** 0.0040 .

Heads	Probability
0	0.064
1	0.288
2	0.432
3	0.216

15.5 Graded revision questions

LEVEL 1

1 $\frac{1}{26}$

2 $\frac{4}{9}$

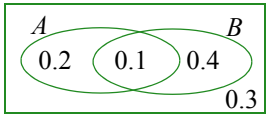
3 $\frac{1}{8}$

4 $\frac{2}{6} = \frac{1}{3}$

5 $\frac{1}{4}$

6 $\frac{1}{2}$

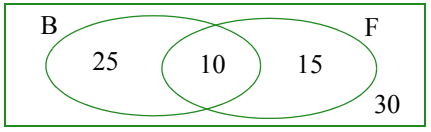
LEVEL 2

1 

a 0.7 **b** 0.5 **c** 0.4

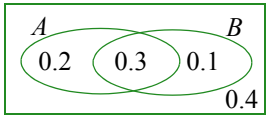
2 **a** 16 **b** $\frac{1}{16}$ **c** $\frac{3}{8}$ **d** $\frac{5}{16}$

3 **a** $\frac{1}{81}$ **b** $\frac{1}{72}$

4 **a** 

b $\frac{1}{8}$ **c** $\frac{5}{16}$ **d** $\frac{3}{8}$

LEVEL 3

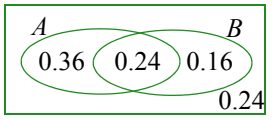
1 

a $\frac{3}{4}$ **b** $\frac{3}{5}$ **c** 0.2 , 0.3 ; the events are not independent.

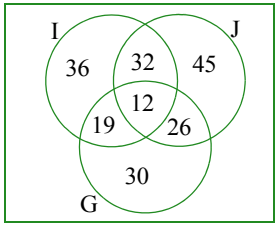
2 **a** $\frac{4}{51}$ **b** No **c** $\frac{1}{13}$

3 **a i** Yes **ii** $\frac{9}{64}$ **iii** $\frac{1}{4}$ **b i** No **ii** $\frac{3}{28}$ **iii** $\frac{3}{14}$

LEVEL 4

1 

a 0.76 **b** 0.24 **c** 0.6

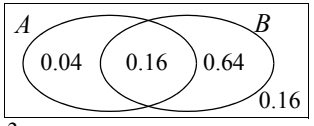
2 **a** 

b $\frac{3}{50}$ **c** $\frac{77}{200}$ **d** $\frac{44}{115}$ **e** No, selections made without replacement are always dependent.

15.6 Topic test

1 **a** 0.2 **b** 0.4

2 **a** 0.25 **b** $\frac{3}{51}$ **c** $\frac{25}{51}$

3 **a** 

b 0.84 **c** 0.16 **d** $\frac{1}{5}$

4 **a** $\frac{3}{7}$ (the results are independent) **b** $\frac{864}{2401}$ **c** $\frac{9}{49}$

Exercise 16.1

1	*a* cm	*b* cm	*c* cm	*A*	*B*	*C*
a	3.8	4.1	1.6	67°	90°	23°
b	81.5	98.3	55.0	56°	90°	34°
c	32.7	47.1	33.9	44°	90°	46°
d	1.61	30.7	30.7	3°	90°	87°
e	2.3	2.74	1.49	57°	90°	33°
f	48.5	77.0	59.8	39°	90°	51°
g	44.4	81.6	68.4	33°	90°	57°
h	2.93	13.0	12.7	13°	90°	77°
i	74.4	94.4	58.1	52°	90°	38°
j	71.8	96.5	64.6	48°	90°	42°
k	23.3	34.1	24.9	43°	90°	47°
l	43.1	43.2	2.3	87°	90°	3°
m	71.5	80.2	36.4	63°	90°	27°
n	33.5	34.1	6.5	79°	90°	11°

o	6.1	7.2	3.82	58°	90°	32°
p	29.1	30.0	7.3	76°	90°	14°
q	29.0	29.1	2.0	86°	90°	4°
r	34.5	88.2	81.2	23°	90°	67°
s	24.0	29.7	17.5	54°	90°	36°
t	41.2	46.2	21.0	63°	90°	27°
u	59.6	72.9	41.8	55°	90°	35°
v	5.43	6.8	4.09	53°	90°	37°
w	13.0	19.8	14.9	41°	90°	49°
x	14.0	21.3	16.1	41°	90°	49°
y	82.4	88.9	33.3	68°	90°	22°

2 **a** $2\sqrt{3}$ **b** $5(1+\sqrt{3})$ **c** 4 **d** $2(1+\sqrt{3})$ **e** $\frac{4}{3}(3+\sqrt{3})$ **f** $\sqrt{106}-5$

3 **a** $25(1+\sqrt{3})$ **b** $\frac{40\sqrt{3}}{3}$

Exercise 16.2

1 **a i** 030°T **ii** 330°T **iii** 195°T **iv** 200°T
b i N25°E **ii** S **iii** S40°W **iv** N10°W
2 37.49 m
3 18.94 m
4 37° 18'
5 $\frac{26}{9}$ m/s
6 N58° 33'W, 37.23 km
7 199.82 m
8 10.58 m
9 72.25 m
10 25.39 km
11 5.76 m
12 **a** 3.01 km N, 3.99 km E **b** 2.87 km E 0.88 km S **c** 6.86 km E 2.13 km N
d 7.19 km 253°T
13 524 m

Exercise 16.3

1 **a** 39°48' **b** 64°46'
2 **a** 12.81 cm **b** 61.35 cm **c** 77°57' **d** 60.83 cm **e** 80° 32'
3 **a** 21°48' **b** 42°2' **c** 26°34'
4 **a** 2274 **b** 12.7°
5 251.29 m
6 **a** 103.52 m **b** 35.26° **c** 39.23°
7 **b** 53.43 m **c** 155.16 m **d** 98.37 m
8 **b** 48.54 m
9 **a** $\sqrt{(b-c)^2+h^2}$ **b** $\tan^{-1}\left(\frac{h}{a}\right)$ **c** $\tan^{-1}\left(\frac{h}{b-c}\right)$
d $2(b+c)\sqrt{h^2+a^2}+2a\sqrt{(b-c)^2+h^2}$
10 82.80 m
11 **a** 40.61 m **b** 49.46 m
12 **a** 10.61 cm **b** 75° 58' **c** 93° 22'
13 **a** 1.44 m **b** 73° 13' **c** 62° 11'

Exercise 16.4

1 **a** 1999.2 cm^2 **b** 756.8 cm^2 **c** 3854.8 cm^2 **d** 2704.9 cm^2 **e** 538.0 cm^2
f 417.5 cm^2 **g** 549.4 cm^2 **h** 14.2 cm^2 **i** 516.2 cm^2 **j** 281.5 cm^2 **k** 918.8 cm^2
l 387.2 cm^2 **m** 139.0 cm^2 **n** 853.7 cm^2 **o** 314.6 cm^2
2 69345 m^2
3 $100\pi - 6\sqrt{91}$ cm^2
4 17.34 cm
5 **a** 36.77sq units **b** 14.70 sq units **c** 62.53 sq units
6 52.16 cm^2
7 27° 2'
8 $\frac{(b+a\times\tan\theta)^2}{2\tan\theta}$
9 Area of $\Delta ABC = 61.38\ cm^2$, Area of $\Delta ACD = 101.78\ cm^2$

Exercise 16.5.1

1	*a* cm	*b* cm	*c* cm	*A*	*B*	*C*
a	13.3	37.1	48.2	10°	29°	141°
b	2.7	1.2	2.8	74°	25°	81°
c	11.0	0.7	11.3	60°	3°	117°
d	31.9	39.1	51.7	38°	49°	93°
e	18.5	11.4	19.5	68°	35°	77°
f	14.6	15.0	5.3	75°	84°	21°
g	26.0	7.3	26.4	79°	16°	85°
h	21.6	10.1	28.5	39°	17°	124°
i	0.8	0.2	0.8	82°	16°	82°
j	27.7	7.4	33.3	36°	9°	135°
k	16.4	20.7	14.5	52°	84°	44°
l	21.4	45.6	64.3	11°	24°	145°
m	30.9	27.7	22.6	75°	60°	45°
n	29.3	45.6	59.1	29°	49°	102°
o	9.7	9.8	7.9	65°	67°	48°
p	21.5	36.6	54.2	16°	28°	136°
q	14.8	29.3	27.2	30°	83°	67°

r	10.5	0.7	10.9	52°	3°	125°
s	11.2	6.9	17.0	25°	15°	140°
t	25.8	18.5	40.1	30°	21°	129°

Exercise 16.5.2

1	*a*	*b*	*c*	A°	B°	C°	c*	B*°	C*°
a	7.40	18.10	21.06	20.00	56.78	103.22	12.95	123.22	36.78
b	13.30	19.50	31.36	14.00	20.77	145.23	6.49	159.23	6.77
c	13.50	17.00	25.90	28.00	36.24	115.76	4.12	143.76	8.24
d	10.20	17.00	25.62	15.00	25.55	139.45	7.22	154.45	10.55
e	7.40	15.20	19.55	20.00	44.63	115.37	9.02	135.37	24.63
f	10.70	14.10	21.41	26.00	35.29	118.71	3.94	144.71	9.29
g	11.50	12.60	22.94	17.00	18.68	144.32	1.16	161.32	1.68
h	8.30	13.70	18.67	24.00	42.17	113.83	6.36	137.83	18.17
i	13.70	17.80	30.28	14.00	18.32	147.68	4.27	161.68	4.32
j	13.40	17.80	26.19	28.00	38.58	113.42	5.24	141.42	10.58
k	12.10	16.80	25.63	23.00	32.85	124.15	5.30	147.15	9.85
l	12.00	14.50	24.35	21.00	25.66	133.34	2.72	154.34	4.66
m	12.10	19.20	29.34	16.00	25.94	138.06	7.57	154.06	9.94
n	7.20	13.10	19.01	15.00	28.09	136.91	6.30	151.91	13.09
o	12.20	17.70	23.73	30.00	46.50	103.50	6.93	133.50	16.50
p	9.20	20.90	27.97	14.00	33.34	132.66	12.59	146.66	19.34
q	10.50	13.30	21.96	20.00	25.67	134.33	3.03	154.33	5.67
r	9.20	19.20	26.29	15.00	32.69	132.31	10.80	147.31	17.69
s	7.20	13.30	18.33	19.00	36.97	124.03	6.82	143.03	17.97
t	13.50	20.40	25.96	31.00	51.10	97.90	9.01	128.90	20.10

2 None of the triangles is possible with the given data.

Exercise 16.5.3

1 30.64 km
2 4.57 m
3 476.4 m
4 201°47'T
5 222.9 m
6 **a** 3.40 m **b** 3.11 m
7 **b** 1.000 m **c** 1.714 m
8 **a** 51.19 min **b** 1 hr 15.96 min **c** 14.08 km
9 $4886 10.906 m

Exercise 16.5.4

1	*a* cm	*b* cm	*c* cm	*A*	*B*	*C*
a	13.5	9.8	16.7	54°	36°	90°
b	8.9	10.8	15.2	35°	44°	101°
c	22.8	25.6	12.8	63°	87°	30°
d	21.1	4.4	21.0	85°	12°	83°
e	15.9	10.6	15.1	74°	40°	66°
f	8.8	13.6	20.3	20°	32°	128°
g	9.2	9.5	13.2	44°	46°	90°
h	23.4	62.5	58.4	22°	89°	69°
i	10.5	9.6	15.7	41°	37°	102°
j	21.7	36.0	36.2	35°	72°	73°
k	7.6	3.4	9.4	49°	20°	111°
l	7.2	15.2	14.3	28°	83°	69°
m	9.1	12.5	15.8	35°	52°	93°
n	14.9	11.2	16.2	63°	42°	75°
o	2.0	0.7	2.5	38°	13°	129°
p	7.6	3.7	9.0	56°	24°	100°
q	18.5	9.8	24.1	45°	22°	113°
r	20.7	16.3	13.6	87°	52°	41°
s	14.6	22.4	29.9	28°	46°	106°
t	7.0	6.6	9.9	45°	42°	93°
u	21.8	20.8	23.8	58°	54°	68°
v	1.1	1.7	1.3	41°	89°	50°
w	1.2	1.2	0.4	85°	76°	19°
x	23.7	27.2	29.7	49°	60°	71°
y	3.4	4.6	5.2	40°	60°	80°

Exercise 16.5.5

1 **a** 10.14 km **b** 121°T
2 7° 33'
3 4.12 cm
4 57.32 m
5 315.5 m
6 **a** 124.3 km **b** W28°47'S

Exercise 16.6 Miscellaneous questions

1 39.60 m, 52.84 m
2 30.2 m
3 54°,42°, 84°
4 37°
5 028°T
6 108.1 cm
7 **a** 135° **b** 136.1 cm
8 41°, 56°, 83°
9 **a** 158° left **b** 43.22 km
10 264 m
11 53.33 cm
12 186 m

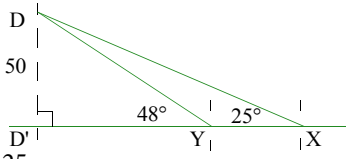
13 50.12 cm
14 5.17 cm
15 **a** 5950 m **b** 13341 m **c** 160° **d** 243°
16 **a** 20.70° **b** 2.578 m **c** 1.994 m^3
17 **a** 4243 m^2 **b** 86 m **c** 101 m
18 **a** 28.28 cm **b** 34.64 cm **c** 35°16′ **d** 45°
19 **a** 9.33cm (approx.) **b** 77°54′ **c** 68°52′
20 **b** 182.08 m
21 **a** 5 **b** 5 **c** 68°54′
22 **b i** $x = \sqrt{288} \approx 16.97$ **ii** 62°
23 **a** 22°35′ **b** 39°48′

16.7 Graded revision questions

LEVEL 1
1 19.01 cm
2 56 cm
3 12.01 cm
4 2.01 cm
5 1.56 cm
6 27°
7 **7.** 56°
8 53°
9 **a** $8\sqrt{2} \approx 11.31$ cm **b** $\theta = 35°16'$
10 **a** $\therefore AC = 5\sqrt{2} = 7.07$ **b** 76°44′
LEVEL 2
1 3.56 cm
2 1.41 cm
3 7.27 cm
4 23.7 cm
5 4.92 cm
6 6.03 cm
7 50.86 cm
8 119.94 cm
9 **a** 264.58 cm **b** 20°42′ **c** 45°
10 25° 35′
11 6.93 cm
12 5.92 cm
13 **a** 9.11 **b** 69°17′
LEVEL 3
1 $\therefore x = \sqrt{3}$
2 $\theta = 55°$. The other angle is = $180° - 55° = 125°$.
3 $c = 1.95$ cm A = 66°, B = 180 – 66 – 63 = 51°
4 0.5
5 ~ 4.35*a*.
6 17°36′
7 **a** 5*r* **b** 12.5 cm
8 70.71 m
9 **a** ~ 11.70 cm **b** 7.70 cm
10 **a** 0.98 m **b** 54°44′
LEVEL 4
1 101°. This is a true bearing. An alternative answer is east 11° south.
2 2.52 m/s to 3 S.F.
3 **a** $AD = \frac{h}{\tan\alpha}$ **b** $\therefore AD = x - \frac{h}{\tan\beta}$ **c** $h = \frac{x\tan\alpha\tan\beta}{\tan\alpha + \tan\beta}$
4 $\alpha = 70°32'$
5 From B $\theta = \tan^{-1}\left(\left(\sqrt{1+\left(\frac{b}{a}\right)^2}\right)\tan\alpha\right)$ From D: $\beta = \tan^{-1}\left(\left(\sqrt{\left(\frac{a}{b}\right)^2+1}\right)\tan\alpha\right)$.
6 $r + \sqrt{2}r = (1+\sqrt{2})r$.

16.8 Topic test

1 72.1 m
2 $a = 4.2727$ to 5 S.F.
3 21.52cm^2 to 4 S.F.
4 69° to the nearest degree
5



$XY = D'X - D'Y = 50\left(\frac{1}{\tan 25°} - \frac{1}{\tan 48°}\right) = 62.21$

6 25 cm
7 **a** 51°1′ **b** 47°52′ **c** 77°57′

Exercise 17.1

1 **a** 0.6915 **b** 0.9671 **c** 0.9474 **d** 0.9965 **e** 0.9756 **f** 0.0054 **g** 0.0287 **h** 0.0594 **i** 0.0073 **j** 0.8289 **k** 0.6443 **l** 0.0823
2 **a** 0.0360 **b** 0.3759 **c** 0.0623 **d** 0.0564 **e** 0.0111 **f** 0.2902 **g** 0.7614 **h** 0.0343 **i** 0.6014 **j** 0.1450 **k** 0.9206 **l** 0.2668 **m** 0.7020 **n** 0.9132 **o** 0.5203

Exercise 17.2

1 **a** 0.0228 **b** 0.9332 **c** 0.3085 **d** 0.8849 **e** 0.0668 **f** 0.9772
2 **a** 0.9772 **b** 0.0668 **c** 0.6915 **d** 0.1151 **e** 0.9332 **f** 0.0228
3 **a** 0.3413 **b** 0.1359 **c** 0.0489
4 **a** 0.8413 **b** 0.4332 **c** 0.7734
5 **a** 0.1151 **b** 0.1039 **c** 0.1587
6 **a** 0.1434 **b** 0.6595
7 **a** –1.2816 **b** 0.2533
8 **a** 58.2243 **b** 41.7757 **c** 59.80
9 39.11
10 9.1660
11 42%
12 0.7021

13 **a** 0.2903 **b** 0.4583 **c** 0.2514

Exercise 17.3 Miscellaneous questions

1 23%
2 0.5
3 11%
4 5%
5 14%
6 1.8
7 252
8 0.1517
9 0.3821
10 0.22
11 322
12 0.1545
13 7
14 87
15 **a i** 0.0062 **ii** 0.0478 **iii** 0.9460 **b** 0.0585
16 **a** \$5.11 **b** \$7.39
17 **a** 0.0062 **b i** 0.7887 **ii** 0.0324 **c** \$1472

17.4 Graded revision questions

LEVEL 1
1 **a** 0.8160 **b** 0.9388 **c** 0.7258
LEVEL 2
1 **a** 0.6827 **b** 0.1359 **c** 0.3934
2 **a** 0.1587 **b** 0.6827 **c** 0.1359
LEVEL 3
1 **a** 0.1908 **b** 0.4754 **c** 16.88
2 **a** 0.1434 **b** 0.6595
3 **a** 0.2425 **b** 0.8413 **c** 0.5050
LEVEL 4
1 9.1660
2 **a** $\mu = 66.86$, $\sigma = 10.25$ **b** \$0.38S
3 **a** $\mu = 37.2$, $\sigma = 28.2$ **b** 20 19.9
4 **a i** 0.3446 **ii** 0.2347 **b i** 0.3339 **ii** 0.3852 **c** 0.9995

17.5 Topic test

1 **a** 0.691 **b** 0.383 **c** 0.444 **d** 0.641 **e** 0.352 **f** 0.106 **g** 0.090
2 **a** 0.533 **b** 0.662 **c** 0.338 **d** 0.136 **e** 0.595 **f** 0.174
3 0.328, ~480 000

Exercise 18.1

1 **a** 38 cm^2 **b** 50 cm^2 **c** 250 cm^2
2 **a** i 11 000 cm^2 **b** 187.5 cm^2 **c** 147 cm^2 **b** i 450 cm **ii** 58.31 cm **iii** 70 cm
3 **a** 30 cm^2 **b** 40 cm^2 **c** 31.43 cm^2
4 **a i** 85 cm^2 **ii** 13.5 cm^2 **iii** $2\sqrt{3}x^2$ cm^2 **b i** 40.2 cm **ii** 17.16 cm **iii** $8x$ cm
5 **a** 4π cm^2 **b** 3π cm^2 **c** $\frac{\pi}{2}$ cm^2
6 **a** 6.5π cm^2 **b** 27.99 cm^2
7 **a** 4.19 cm^2 **b** 8.19 cm
8 **a** 21.46 cm^2 **b** 31.41 cm
9 294.52 cm^2
10 **a i** 13.75 m **ii** 10.33 m^2 **b** \$129.13
11 **a** 109.90 cm **b i** 68.94 cm **ii** 3788.44 cm^2 **iii** 2233.68 cm
12 **a** 5 **b** 4 **c** 20
13 **a** 6.40 cm **b** 102.68° **c** 20 cm^2 **d** 92.07 cm^2
14 **a** 146.21 cm **b** 1513.46 cm^2

Exercise 18.2

1 **a** 220 **b** 580 **c** 189.25
2 **a** 64.87 **b** 98.49 **c** 725.40
3 a 31π cm^2 or 97.39 cm^2 b 192π cm^2 or 603.19 cm^2

Exercise 18.3

1 **a** 24 cm^3 **b** 80 cm^3 **c** 231 cm^3
2 **a** 51.96 cm^3 **b** 115.93 cm^3 **c** 229.77 cm^3
3 **a** 5 **b** 150 cm^2 **c** Increase by 72.8%
4 **a i** 3 **ii** 113.10 cm^2 **iii** 1 **b** $\frac{1}{3}R$
5 **a** 10.05 m^3 **b** 18.10 m^3 **c** 2.0 m^3 **d** 1.88 m^3
6 **b** $h = \sqrt{51}$ **c i** 57.13 m^3 **ii** 99.48 m^2
7 **a** 7.4220 m^3 **b** \$138.05 **c** 49.48 m^2
8 **a** 5 **b i** $5(x-10)^2$ cm^3 **ii** $x^2 - 100$ **c** 21.18
9 216 cm^3
10 **a** 25.13 cm^3 **b** 2.93 cm
11 **a i** 20 **ii** $\frac{9500}{3}\pi$ cm^3 **b** $r = 6.41$ **c i** 41.23 cm **ii** 61.85 cm **iii** 2640.16 cm^2

Exercise 18.4 Miscellaneous questions

1 **a i** 22 cm **ii** 30.25 cm^2 **b i** Not enough info! **ii** 28 cm^2 **c i** Not enough info! **ii** 262.5 cm^2 **d i** not enough info! **ii** 27 cm^2 **e i** 38 cm **ii** 66 cm^2 **f i** 6π cm **ii** 3π cm^2
2 **a** 37.5 cm^2 **b** 2π cm^2 **c** 13.75 cm^2 **d** 60 cm^2
3 **a** 248 cm^2 **b** 238π cm^2 **c** $(196 + 14\sqrt{596})\pi$ cm^2 **d** $100(1 + \sqrt{10})$ cm^2
4 **a** 399π cm^2 **b** $(24.5 + 3.5\sqrt{112.25})\pi$ cm^2
5 **a** 63 cm^3 **b** 500 cm^3 **c** $\frac{5}{3}\pi$ cm^3
6 **a** $36\sqrt{216}$ cm^3 **b** Vol = surface area × width = $8.5w$ cm^3, where w is the width of shape.

18.5 Graded revision questions

LEVEL 1

1 **a** $4(7-2x)$ cm **b** $7x+7$ cm **c** $8+2\pi$ cm

2 **a** 2.52 cm^2 **b** $4\pi + 20$ cm^2

3 27:1

LEVEL 2

1 12 cm^2

2 $120 + 12\sqrt{13.75}$ $cm^2 \sim 164.50$ cm^2

3 **a** 1 **b** The smaller fish balls; diff = $(2000\pi - 400\pi)$ $cm^2 = 1600\pi$ cm^2

LEVEL 3

1 $\frac{512}{3}$ cm^3

2 $r = 6$, Area = 118.90 cm^2

3 **a** 0.5 **b** 10 cm **c** 1000π cm^3

LEVEL 4

1 **a** 16940π cm^3 **b** 2618π cm^2

2 $\frac{1}{6}V$ cm^3

3 1:2

4 $\frac{13}{4}$ m^3

18.6 Topic test

1 6 cm

2 **a** 528 cm^2 **b** 480 cm^2

3 **a** $1:\sqrt{3}$ **b** $\sqrt{3}:2$

4 $\frac{1}{6}x^3$ cm^3

5 **a i** $2r$ **ii** $4\pi r^2$ **b** They are equal.

Exercise 19.1

1 **a i** 14 500 **ii** 2 000 **b** 305 (304.5)

2 Sample size is large but may be biased by factors such as the location of the catch. Population estimate is 5000.

3 **a i** 1500 **ii** 120 **b** 100 **c** 1 000

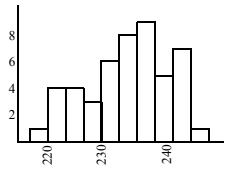
4 **a, c** numerical, **b, d, e** categorical

5 **a, d** discrete, **b, c, e** continuous

Exercise 19.2

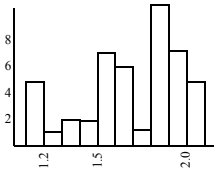
1

218–220	221–223	224–226	227–229	230–232	233–235	236–238	239–241	242–244	245–247
1	4	4	3	6	8	9	5	7	1



2

1.1–1.2	1.2–1.3	1.3–1.4	1.4–1.5	1.5–1.6	1.6–1.7	1.7–1.8	1.8–1.9	1.9–2.0	2.0–2.1
5	1	2	2	7	6	1	12	7	5



3 Set A Mode = 29.1 Mean = 27.2 Median = 27.85
Set B Mode = 9 Mean = 26.6 Median = 9. Set B is much more spread out than set A and although the two sets have a similar mean, they have very different mode and median.

Exercise 19.3

1 Mode = 236–238 g, Mean = 234 g, Median = 235 g

2 Mode = 1.8–1.9 g, Mean = 1.69 g, Median = 1.80 g

3 Set A Mode = 29.1, Mean = 27.2, Median = 27.85; Set B Mode = 9, Mean = 26.6, Median = 9.

4 **a** \$27522 **b** \$21025 **c** Median

5 **a** \$233 300 **b** \$169 000 **c** Median

6 **a** 14.375 **b** 14.354

Exercise 19.4

1 **a** Sample A Mean = 1.99 kg; Sample B Mean = 2.00 kg
b Sample A Sample std = 0.0552 kg; Sample B Sample std = 0.1877 kg
c Sample A Population std = 0.0547 kg; Sample B Population std = 0.1858 kg

2 **a** 16.41 **b** 6.84

3 Mean = 49.97, Std = 1.365

Exercise 19.5

1 **a** Med = 5, Q1 = 2, Q3 = 7, IQR = 5 **b** Med = 3.3, Q1 = 2.8, Q3 = 5.1, IQR = 2.3
c Med = 163.5, Q1 = 143, Q3 =182, IQR = 39
d Med = 1.055, Q1 = 0.46, Q3 = 1.67, IQR = 1.21
e Med = 5143.5, Q1 = 2046, Q3 = 6252, IQR = 4206

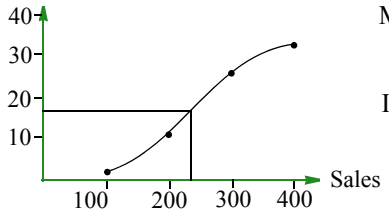
2 **a** Med = 3, Q1 = 2, Q3 = 4, IQR = 2 **b** Med = 13, Q1 = 12, Q3 = 13, IQR = 1
c Med = 2, Q1 = 2, Q3 = 2.5, IQR = 0.5

d Med = 40, Q1 = 30, Q3 = 50, IQR = 20
e Med = 20, Q1 = 15, Q3 = 22.5, IQR = 7.5

3 **a** \$84.67 **b** \$147.8 **c** \$11 **d** Q1 = \$4.50, Q3 = \$65 IQR = \$60.50
e Median and IQR.

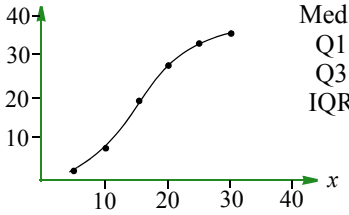
4 **a** 2.35 **b** 1.25 **c** 2 **d** Q1 = 1, Q3 = 3, IQR = 2

5 **a** \$232 **b** \$83 **c–e**



Med = \$220
Q1 = \$160
Q3 = \$310
IQR = \$150

6



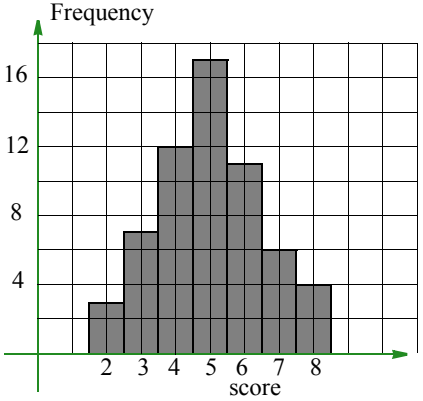
Med = 14
Q1 = 10
Q3 = 19
IQR = 9

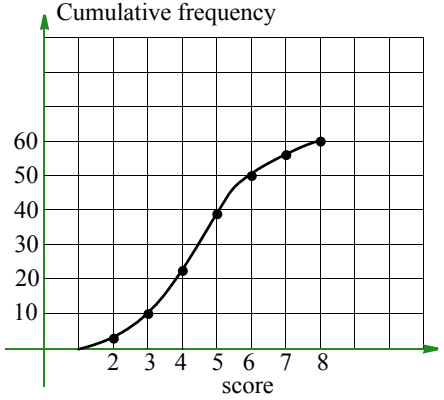
Exercise 19.6 Miscellaneous questions

1 **a** Sample–100 randomly selected patients, population – all suffering from AIDS
b Sample–1000 working aged people in NSW, population – all working aged people in NSW
c Sample – John's I.B Higher Maths class, population – all seniors at Nappa Valley High School.

2 Discrete: a, b, d; Continuous: c, e, f, g.

3 **b**





4 Suggested answers only: **a** 200–224; 225–249; 250–274; . . . 575–599
b 100–119; 120–139; . . . 400–419. **c** 440–459; 460–479; . . . 780–799.

5 Make use of your graphics calculator.

6 **a** 16 **b** graphics calculator **c** 15.23 **d** 15.5 **e** Q1 = 14, Q3 = 17
f 15.87 (2 d.p.)

7 **a** 30–34 **b** graphics calculator **c** 30.4 **d** 32 (approx.)

8 **b** 215.5 **c** 216.2

9 48.17

10 **a** Q1~ 35, Q3~ 95 **b** ~ 104 **c** 60% **d** 67.15

11 range = 19, s = 5.49

12 5.8; 1.50

13 17.4; $s_n = 3.12$ $s_{n-1} = 3.18$

14 **a** 6 **b** 7 **c** Q1 = 5, Q3 = 7 **d** 2 **e** 6.15 **f** 1.61

15 $s_n = 18.8$, $s_{n-1} = 19.1$

16 14.18

19.7 Graded revision questions

LEVEL 1

1 **a** 1 b 13 c 1.3 d 40%

2 **a** 1 b 3 c 10–14 and 20–24

3 75

LEVEL 2

1 **a** 2.09 **b** 13.7 **c** 4.05 **d** 9.28 **e** 52.6

2 **a** 4 **b** 3 **c** 12 **d** 13 **e** 16 and 17

3 **a** 17 **b** 31 **c** 4.2 **d** 49.1 **e** 0.8

LEVEL 3

1 **a** 2 and 7 **b** 22 and 82 **c** 23.5 and 84.2
d 5.5 and 20 (take the mid-points between 5 and 6, and 19 and 21)
e 3.4 and 6.8

2 **a** 2.57 **b** 30.1 **c** 2.91 **d** 279 **e** 0.296

LEVEL 4

1 The median as this is the measure that is least affected by the small number of outliers (the luxury homes).

2 **a** Frequency table required **b** mean = 6.35 g **c** 2.615 g

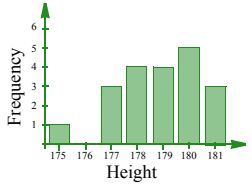
3 **a** 50 **b** 5 **c** 2 **d** The mean is likely to be about 5 as there is a positive skew.
e The range is 9 – 2 = 7. A complete distribution usually covers 6 standard deviations. On this basis the standard deviation might be expected to be about 7/6 or 1.2 marks.

19.8 Topic test

1 **a**

Height	175	176	177	178	179	180	181
Frequency	1	0	3	4	4	5	3

b



c Height 175–179; 180–184 Frequency 12 8 First method is best.
d Mode = 180 cm, Mean = 178.85, Median = 179 cm
e 1.558 cm, Quartiles are 178 and 180 cm, Interquartile range = 2 cm
f

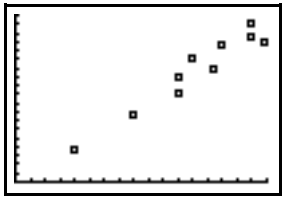
17	5	7	7	7	8	8	8	8	9	9	9	9
18	0	0	0	0	0	1	1	1				

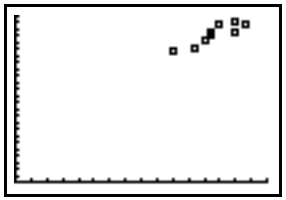
key: 17|5 = 175

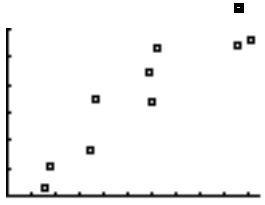
2 **a** 19 **b** 418.42105 ≈ \$420 **c** \$450
d These are probably part-time or casual workers. e The mean

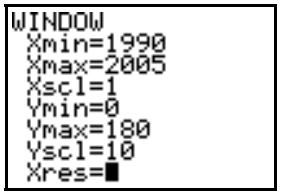
Exercise 20.1

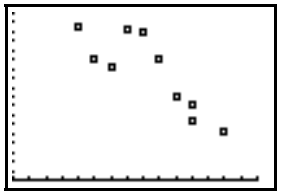
1 **a i** Increasing, positive **ii** approx. linear **iii** mild (to weak)
b i No association **ii–iii** 0
b i Increasing, positive **ii** linear **iii** very strong
d i Increasing, positive **ii** square root **iii** mild (strength not appropriate as it is a non–linear relationship!)
e i Decreasing, negative **ii** exponential **iii** mild (stength not appropriate as it is a non–linear relationship!)
f i Decreasing **ii** approx. linear **iii** mild

2 **a**  **b** Positive association, linear, strength: very strong

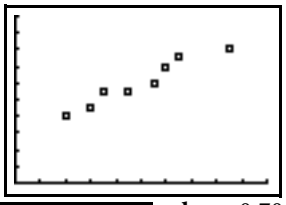
3 **a**  **b** Positive association, linear, strength: very strong

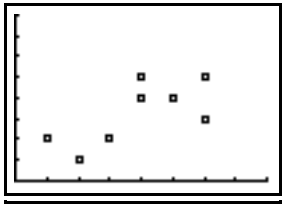
4 
Data displays a strong positive association. Increase in lead content can be attributed to increase in traffic flow.

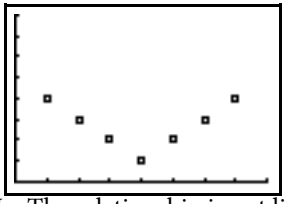
5 


Worksafety policy has had desired effect, i.e. number of accidents has decreased. Data displays a strong negative association.

Exercise 20.2

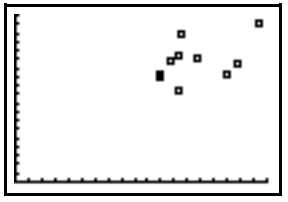
1 **a** $r = 0.96$ **b** 

2 **a**  **b** $r = 0.70$ (aasumed linear)

3 **a**  **b** $r = 0$. No, not linear!

4 No. The relationship is not linear.

5 **a i** 64% **ii** 81% **b i** 51% **ii** 64%

6 **a**  **b** $r = 0.45$

7 3

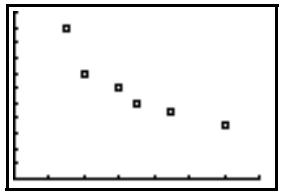
8 ± 0.922

9 82%

10 **b** $r = 0.7715$ There is strong evidence to suggest that a student who does well in maths will also do well in Biology. **c** Values of r are the same.

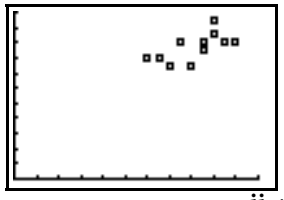
Exercise 20.3

1 a i



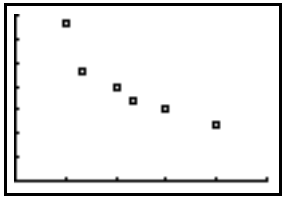
ii $y = -1.33x + 21.11$

b i

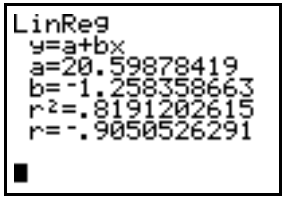


ii $y = 0.64x + 6.94$

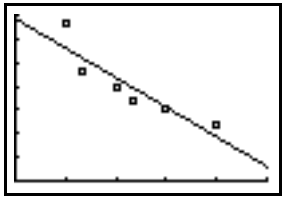
2 a i



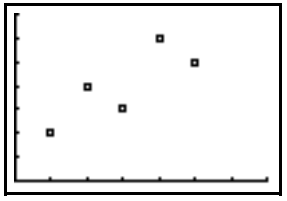
ii $y = 20.6 - 1.26x$



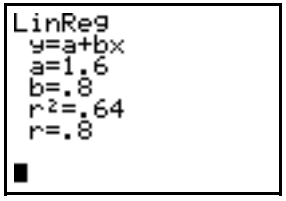
iii



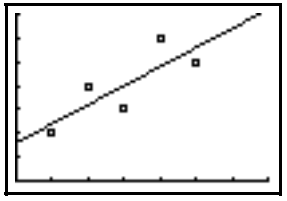
b i



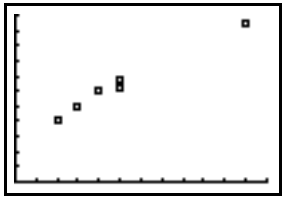
ii $y = 1.6 + 0.8x$



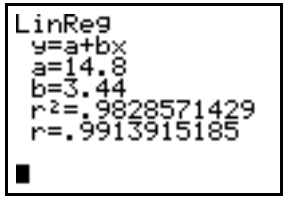
iii



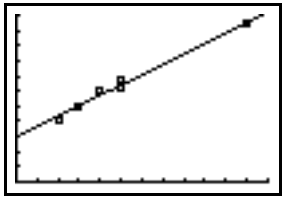
c i



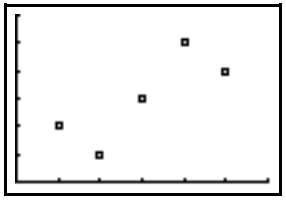
ii $y = 14.8 + 3.44x$



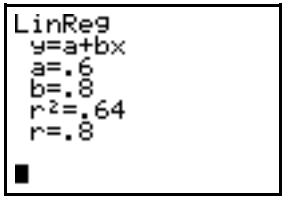
iii



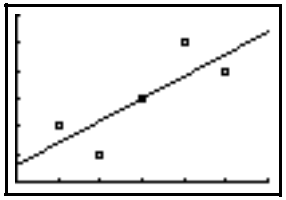
d i



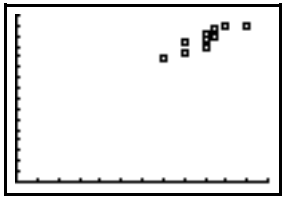
ii $y = 0.6 + 0.8x$



iii

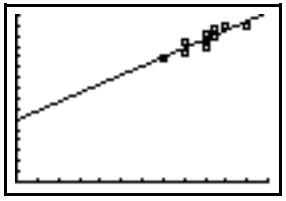


3 a

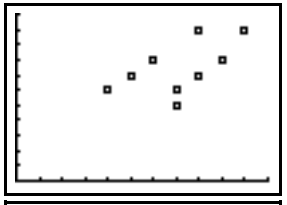


b $r = 0.891$
c 79.4%
d $y = 29.76 + 2.15x$

e

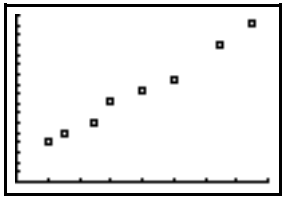


4 a



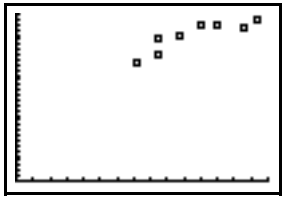
b $r = 0.553$
c i $y = 40 + 0.5x$
ii $x = 24.1 + 0.61y$

5 a



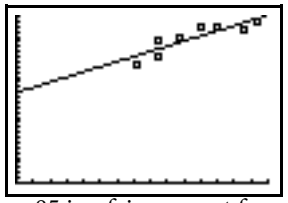
b Based on the scatter diagram, there is a definite linear relationship. Therefore, owner is justified.
c i $r = 0.99$ **ii** $C = 4.19 + 1.82w$
d i 20.57, i.e. 21 **ii** 95.19, i.e. 95
iii From **ii**, serving 95 people per hour is unrealistic.

6 a



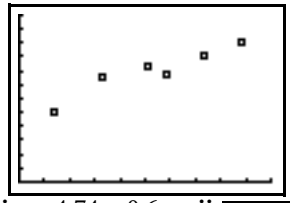
b Scatter diagram shows a linear relationship. Therefore statistic is appropriate, $r = 0.877$.

c i $y = 89.50 + 1.02x$ **ii**



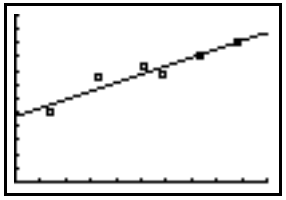
d i 135.6 **ii** 176.5 **iii** $x = 85$ is a fair way out from the set of values used to obtain the regression line

7 a

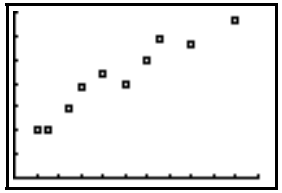


b Scatter diagram shows a linear relationship. Therefore statistic is appropriate, $r = 0.945$

c i $y = 4.74 + 0.6x$ **ii**



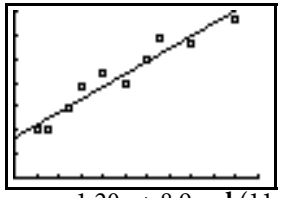
d i 8.63 **ii** 10.73



8 a

b Linear trend exists, $r = 0.96$

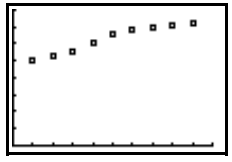
c i $y = 2.68x + 16.86$ **ii**



d i 27.57 **ii** 57.03

9 a $r = 0.8384$ **b** 70.29% **c** $y = 1.20x + 8.9$ **d** (11, 22.1), (24.45, 38), (27, 41.3), (60.08, 81). The equation is used to predict y from x-values, not x from y-values. We would need to find the regression of x on y.

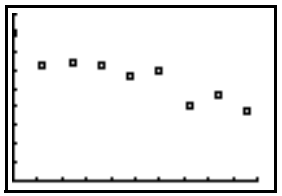
10 a



b i $r = 0.97$ **ii** 222 **c** $M = 0.2967T + 48.28$

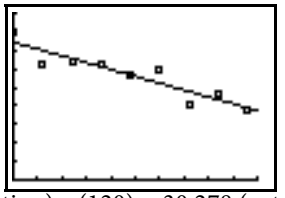
11 Remains the same.

12 a



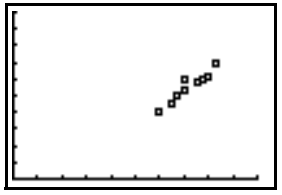
b −0.93

c i $y = -0.37x + 74.44$ **ii**



d $y(40) = 59.722$ (interpolation); $y(120) = 30.278$ (extrapolation)

Exercise 20.4 Miscellaneous questions



1 a **b** $y = 0.57x - 26.2$

c 0.9388 Because of the strong positive linear association, and the high r value, we can say that the taller the student the greater their weight.

2 0.057

3 B

4 a 0.8 **b** strong positive relationship

5 1.5

6 a 0.78 **b i** P = 1.07M − 12.91 **ii** 73% **c i** M = 0.77E + 27.14 **ii** 100

iii Extrapolated. Continued linear trend highly likely. Therefore confident

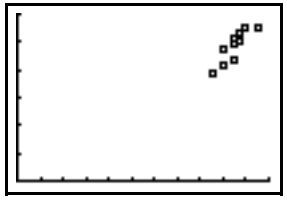
d Find regression equation of E on M, then use M = 90 into this new equation.

7 a positive **b** linear **c** very strong

8 a $\bar{x} = 20.57, \bar{y} = 31.86$ **b** 0.9645 **c** $y = 1.68x - 2.7$

9 a $y = -1.75x + 64.67$ **b** 22.67 **c** −11.12

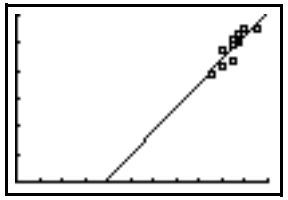
10 a i



ii = 0.8908

b $r^2 = 1.7935$, that is, 79.35%

c $y = 2.15x - 33.28$

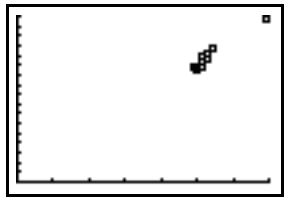
d 

e $x = 37$, $y = 46.35$; Expenditure is \$4635

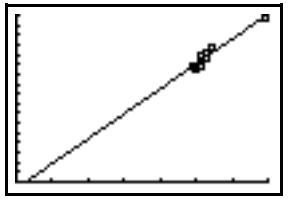
11 a i 4.4; 2.02 **ii** 14.06; 2.92 **b** $b = 0.4895$; $r = 0.34$

c $r^2 = (0.3397)^2 = 0.1154$

d Regression equation is $(y - 14.06) = 0.4895(x - 4.4) \therefore$ when $x = 3.5$, $y = 13.63$



12 a i **ii** $r = 0.9629$ **b** $y = 0.635x - 33.815$

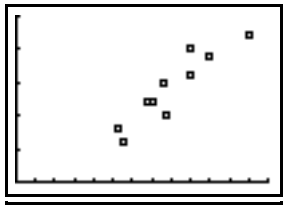
c 

d When $x = 1040$, $y = 626.59$. The carcass weighs 626.59 lbs.

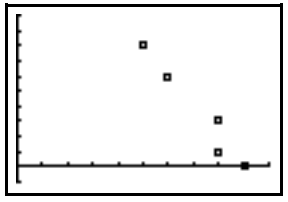
20.5 Graded revision questions

LEVEL 1

1 a b +ve direction, strong linear relationship.



2 –ve direction, strong and linear relationship.



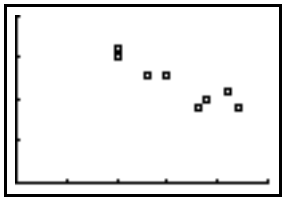
LEVEL 2

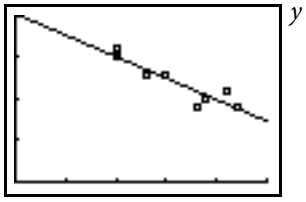
1 $r = 0.9213$

2 $r = 0.9686$

LEVEL 3

1 As the scatter plot indicates, there exists a strong positive linear relationship, r measures the strength of the relationship, whereas, r^2 represents the proportion of the variation in y which can be attributed to the variation in x (in this case it would be 93.81%).

2 a 

b 

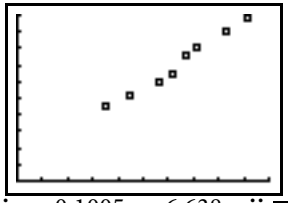
$y = -0.52x + 20.31$

c $x = 14$, $y = 13.03$

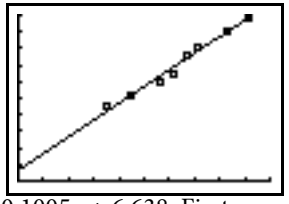
3 a $b = 0.4749$ $r = 0.92$ **b** –9.745

4 $r^2 = 0.9 \therefore r = 0.9487$

LEVEL 4

1 a 

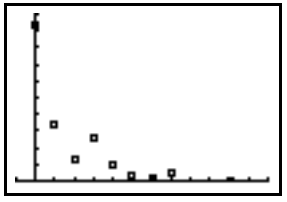
b Yes. Scatter diagram shows a strong +ve linear relationship.

c i $y = 0.1005x + 6.638$ **ii** 

d = 0.993

e $x = 600$, $y = 66.94$

f Yes. But **not** by using $y = 0.1005x + 6.638$. First you need to find the regression of x on y, i.e. $x = by + c$, which is given by $x = 9.808y - 55.76$. Therefore, $x(56) = 493$.

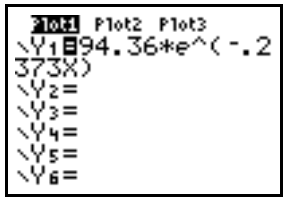
2 a 

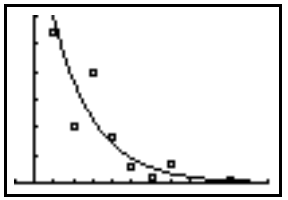
b $Y = -\alpha r + C$ with $Y = lnd$ where , $C = lnk$, we have

$Y = -0.2373r + 4.5471$.

$\Rightarrow k = e^{4.5471} = 94.36$

c i

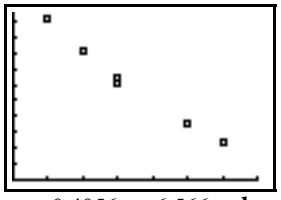




ii $r = 16$, $d = 2.117$

20.6 Topic test

1 a

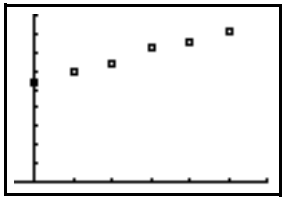


b i –ve **ii** linear **iii** strong **c** $y = -2.97x + 44.28$

2 a $y = 0.4056x + 6.566$ **b** $x = 8$, $y = 9.81$ **c** $s_{xy} = 2.92$

3 $r^2 = 0.80 \therefore r = 0.8944$

4 a



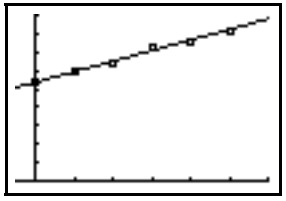
b From the given data:

$6a + 150b = 408$ … (1)

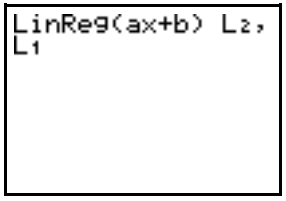
$150a + 5500b = 11200$ … (2)

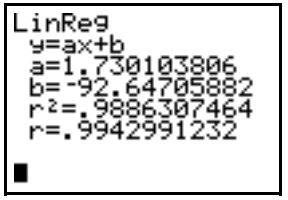
$b = \frac{4}{7} \therefore a = \frac{376}{7}$

c



d $T = 25$, $x = 68$





e i First you need to find the regression of T on x, i.e. $T = bx + a$

ii $T = 1.73x - 92.65$, therefore, $x = 70$, $T = 28.45$

Exercise 21.1

1 a Reject independence (i.e. reject H_0) **b** Accept independence (i.e. accept H_0).

2 Reject H_0

3 Accept H_0

4 Accept H_0 (i.e. independent)

5 a Reject H_0 **b** Reject H_0

6 Reject H_0

7 Accept H_0

8 Accept H_0

9 Reject H_0 at the 5% level of significance.

10 a The treatment has a significant effect, that is, of those treated, a significantly smaller proportion developed the flu than those that were not treated.
b Results are not significant. The significance test used in **a** is accurate because the sample size is large, but in **b** it is inaccurate because the sample size is too small.

11 Yes, $p = 1.178 \times 10^{-7}$

12 Yes, $p = 6.35 \times 10^{-6}$

Exercise 21.2 Miscellaneous questions

1 a

Expected values:

Maths Studies \ Biology	Excellent	Good	Fail
Excellent	17.94	28.86	31.2
Good	31.74	51.06	55.2
Fail	42.32	68.08	73.6

b Biology results are independent of Mathematical Studies results.

c $\chi^2_{calc} = 61.59$ **d i** 4 **ii** 9.488 **iii** Reject H_0. That is, there exists a positive relationship between doing well in Mathematical Studies and Biology.

2 a

Expected values:

Year since vaccination took place \ Severity of smallpox attack	Extreme	Very	Moderate	Light
0 – 20	99	149.96	141.22	104.82
20 – 40	187	283.25	266.75	198
Over 40 or unvaccinated	54	81.79	77.03	57.18

b The severity of smallpox is independent of when vaccination was administered.

c i 6 **ii** $\chi^2_{calc} = 153.46$, reject H_0, i.e., severity of smallpox is dependent on when vaccination was administered.

3 $\chi^2_{calc} = 5.33$, $p = 0.021$. Therefore reject the null hypothesis at 5% level. That is, difference between team performance exists.

4 a

Referendum question

	For	Against
Male	45	55
Female	45	55

b Preference on referendum issue is independent of gender.

c i 18.18 **ii** $p = 2.01 \times 10^{-5}$. Therefore, reject the null hypothesis. That is, gender does play a part in preference on referendum issue.

5 a $a = 52.89$, $b = 165.13$, $c = 53.87$, $d = 3.94$ **b** H_0: Occupation is independent of the amount of schooling received. H_1: Occupation is dependent on the numbers of years spent at school.

c i d.f. = 3, $\chi^2 = 7.81$ **ii** $\chi^2_{calc} = 52.1$ **iii** Reject H_0.

21.3 Graded revision questions

LEVEL 1

1 a

	A1	A2
B1	27.08	22.92
B2	37.92	32.08

b

	A1	A2	A3
B1	17.5	22.5	10
B2	10.5	13.5	6

LEVEL 2

1 a There exists an association. **b** No association.

LEVEL 3

1 a

	Good	Poor
A	6	24
B	14	56

b As $p = 1.27 \times 10^{-5}$, we reject H_0. That is, there is a significant difference between company and absenteeism.

LEVEL 4

1 a **Table 1** Level of self-esteem

	High	Medium	Low
Smoker	21.56	21.56	25.88
Non–smoker	28.44	28.44	34.12

We have assumed that there exists no association between the level of self-esteem and the smoking habits of males.

b $\chi^2_{calc} = 2.00$ **c** p-value = 0.37 and so as $p > 0.05$ we accept H_0. That is, there is no significant difference between self-esteem of smokers as compared to non-smokers.

d $p = 0.242$, therefore still no significant difference (i.e. we still accept H_0).

21.4 Topic test

1 a $a = 30$, $b = 70$, $c = 30$, $d = 70$ **b** $\chi^2_{calc} = 9.52$.

2 $\chi^2_{calc} = 0.488$, $p = 0.921 > 0.05$, therefore accept H_0. That is, there is no significant difference between the type of bread and opinion. Initial assumption was that the categories were independent (so that the expected values table could be constructed).

3 a H_0: Vaccine has no effect on the severity of infectious disease. H_1: Level of severity of infectious disease is dependent on the vaccine having been administered.

b

	High	Medium	Low
Vaccinated	26.53	26.53	30.94
Not vaccinated	33.47	33.48	39.05

c i d.f. = 2, $\chi^2_{critical} = 5.99$ (at 5% sig. level) **ii** $\chi^2_{calc} = 1.73$

iii Accept H_0, i.e. this vaccine has no effect on the severity of the infectious disease.

Exercise 22.1.2

1 **a** 839 **b** 565 **c** 516 **d** 409 **e** 749 **f** 389
2 **a** 1271 **b** 573 **c** 1111 **d** 1646 **e** 500 **f** 486
3 **a** 1745 **b** 395 **c** 1791 **d** 1721 **e** 1303 **f** 1838
4 **a** 31.22 **b** 28.48 **c** 0.57 **d** 20.06 **e** 27.21 **f** 34.41
5 **a** 89 **b** 21 **c** 38 **d** 120 **e** 167 **f** 73

Exercise 22.1.3

1 **a i** ¥143100 **ii** US$982.16 **iii** US$17.84 **b i** ¥183311 **ii** US$1258.14 **iii** US$22.86 **c i** ¥216510 **ii** US$1486 **iii** US$27 **d i** ¥194044 **ii** US$1331.80 **iii** US$24.0 **e i** ¥183597 **ii** US$1260.11 **iii** US$22.89 **f i** ¥228245 **ii** US$1566.54 **iii** US$28.46

2 **a i** 1019000 **ii** 986 **iii** 14 **b i** 1638552 **ii** 1586 **iii** 22 **c i** 1384821 **ii** 1341 **iii** 18 **d i** 1495892 **ii** 1448 **iii** 20 **e i** 1636514 **ii** 1584 **iii** 22 **f i** 1240123 **ii** 1201 **iii** 16

3 **a i** 28710 **ii** 852 **iii** 48 **b i** 45713 **ii** 1356 **iii** 77 **c i** 44788 **ii** 1329 **iii** 75 **d i** 44341 **ii** 1316 **iii** 74 **e i** 42555 **ii** 1263 **iii** 71 **f i** 47467 **ii** 1409 **iii** 79

4 **a i** 40358 **ii** 6447 **iii** 180 **b i** 45620 **ii** 7288 **iii** 203 **c i** 44810 **ii** 7158 **iii** 200 **d i** 45504 **ii** 7269 **iii** 203 **e i** 37764 **ii** 6033 **iii** 168 **f i** 42813 **ii** 6839 **iii** 191

Exercise 22.2.1

1 **a** $804.00 **b** $562.32 **c** $218.40 **d** $279.00 **e** $6,528.00 **f** $812.50
2 **a** $321.60 **b** $515.46 **c** $46.80 **d**. $96.88 **e** $191.25 **f** $187.50
3 **a** 7% **b** 3% **c** 8% **d** 12% **e** 11.8% **f** 9.3%
4 **a** 300 **b** 790 **c** 2000 **d** 365 **e** 1362 **f** 9035 (all marks)
5 **a** 5 **b**. 7 **c** 2 **d** 8 **e** 11 **f** 3.6 (all years)
6 **a** 12 **b** 7 **c** 2 **d** 8 **e** 11 **f** 3.6 (months)

7 Option 1: $I = \frac{6000 \times 7.3 \times 5}{100} = 2190$ francs. Total is 8190 francs.

Option 2: ¥6000 × 153.1 = ¥918600. Interest is $\frac{918600 \times 7.4 \times 5}{100} = 339882$ and the total = ¥1258482. Conversion back to Swiss francs gives $\frac{1258482}{155.7} = 8082.736$ or 8083. Option 1 is better.

Exercise 22.2.2

1 **a** 3682 **b** 8246 **c** 19576 **d** 1521 **e** 5903 **f** 3117
2 **a** 12087 **b** 112108 **c** 128192 **d** 151518 **e** 194298 **f** 48698
3 **a** 14974 **b** 31103 **c** 30083 **d** 32593 **e** 39190 **f** 27899
4 **a** 22122 **b** 28700 **c** 23635 **d** 31155 **e** 29098 **f** 1616
5 A gives $2415.77 B gives $2431.52 and C gives $2411.87 so B is best.
6 14.65%
7 $3952.08

Exercise 22.2.3

1 **a i** $1432.33 **ii** $6432.33 **b** 7.162%
2 **a** $5656.79 **b i** $4856 **ii** $3428
3 14.65%
4 A: $7200 B: $7447.12 C: $7403.89 Therefore choose B.
5 5.90%
6 $8075.20
7 $1428.76
8 $1285.70
9 7.5%
10 **a** Option A **b** $2.52
11 $10,756.96
12 **a** 10.25% **b** 10.38% **c** 10.47%

Exercise 22.2.4

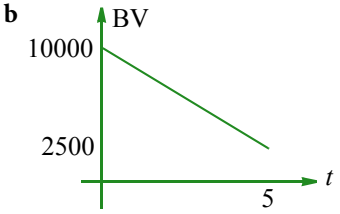
1 **a** 7 years 3 months **b** 7.06 years **c** 7 years
2 $n = 18$; $4\frac{1}{2}$ years
3 **a** 5 years **b** 4 years and 9 months (or 19 quarters)
4 73
5 3 years and 9 months
6 **a** yes, $863.23 **b ii** 8 years three month
7 Kristian: $n = 5.53$. Jørgi $n > 25.498$ i., 6.37 years. Kristian goes 10 months before Jørgi. NB: If they cannot take their money out until the end of the period then Kristian goes 6 months before Jørgi.

Exercise 22.3.1

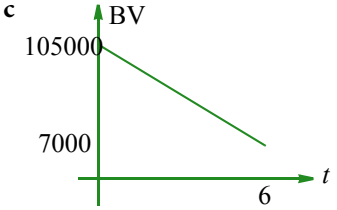
1 $3200
2 $540

Exercise 22.3.2

1 **a** $10000 **b** $5000
2 $3600
3 **a i** $8000 **ii** 4 years **iii** $6000 **iv** $1500 per year **v** $BV = 8000 - 1500t, 0 \le t \le 4$ **vi** $3500 **b i** $400000 **ii** 8 years **iii** $400000 **iv** $50000 per year **v** $BV = 400000 - 50000t, 0 \le t \le 8$ **vi** $250000 **c i** $250000 **ii** 10 yrs **iii** $200000 **iv** $20000 per year **v** $BV = 250000 - 20000t, 0 \le t \le 10$ vi. $190000
4 **a** Table of values using the expression $BV = 10000 - 1500t, 0 \le t \le 5$.
b



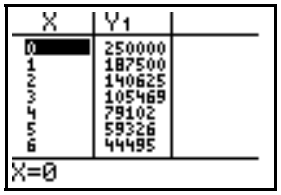
c $2500 **d** $BV_t = 10000 - 1500t, 0 \le t \le 5$
5 **a** $16333.33 per year **b** Table of values using $BV_t = 105000 - \frac{49000}{3}t, 0 \le t \le 6$
c

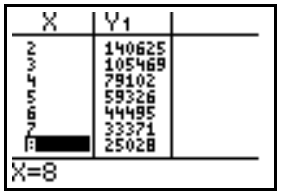


d $BV_t = 105000 - \frac{49000}{3}t, 0 \le t \le 6$
6 Printer B by 3 months and 8 days

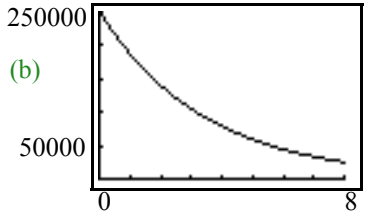
Exercise 22.3.3

1 **a** $8192 **b** $5243
2 **a**

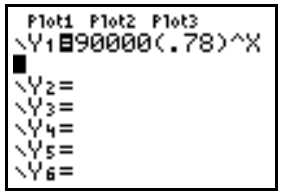


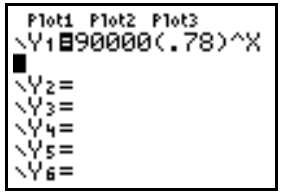


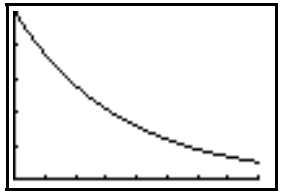
(b)



3 **a i** $70200 **ii** $25984.56 **b** $BV_t = 90000 \times (0.78)^t, 0 \le t \le 10$ **c** $7502.20
d







4 **a**

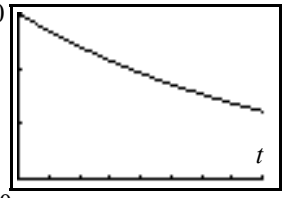
X	Y1
0	15000
.5	13416
1	12000
1.5	10733
2	9600
2.5	8586.5
3	7680

X=0

X	Y1
1	12000
1.5	10733
2	9600
2.5	8586.5
3	7680
3.5	6869.2
4	6144

X=4

b 15000



c $BV = 15000(0.8)^t, 0 \le t \le 4$
d **i** \$9600 **ii** \$6869.20
e \$6144.00

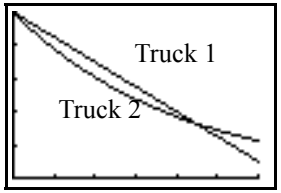
5 \$4507.50

6 **a**

X	Y1	Y2
0	50000	50000
1	42500	39000
2	35000	30420
3	27500	23728
4	20000	18508
5	12500	14436
6	5000	11260

X=0

b



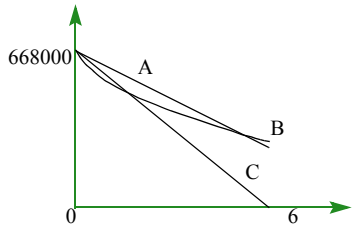
c 4th year
d Truck 1: \$5000
Truck 2: \$11260

7 7.9983 ~ 8 years
8 **a** 7 years **b** \$12801.50

Exercise 22.3.4

1 **a** \$37521.60 **b** 150000 km
2 **a** \$1624 **b** \$12776 **c** 10 years 1 month (approx.)
3 \$19305.88
4 **a** \$26400 **b** 11696.75 km
5 **a** \$110000; \$132500; \$150000; \$142000 **b** $\frac{4310}{N}$ years
6 **a** \$38300 **b** \$40170 **c** Approx. 65455
7 **a, b** Construct a table using the following equations: **A**: BV = 668000 – 80160*t*
B: BV = 668000 × $(0.82)^t$ **C**: BV = 668000 – 114270*t*

c



d **i** C **ii** A
e **i** \$267200
ii \$247645
iii \$96650
f **i** C **ii** 5.85 years

Exercise 22.4.1

1 1034
2 21076
3 \$159.26
4 125100
5 252298

Exercise 22.4.2

1 **a** 56.61 **b** 576.65 **c** 36.06 **d** 152.22 **e** 79.08 **f** 287.35 **g** 43.07 **h** 144.84
2 **a** \$825.95 **b** \$178.04 **c** \$72.05 **d** \$146.74 **e** \$219.84 **f** \$402.15 **g** \$58.93
h \$96.24 **i** \$713.31 **j** \$97.16 **k** \$12.88 **l** \$359.75 **m** \$193.42
n \$94.41 **o** \$5.65 **p** \$298.74 **q** \$109.30 **r** \$39.48 **s** \$43.77 **t** \$55.84

Exercise 22.4.3

1 **a** \$837.71 **b** \$1894.10 **c** \$1022.91 **d** \$602.80 **e** \$339.67 **f** \$680.10
g \$1467.17 **h** \$150.94 **i** \$560.68 **j** \$3833.22 **k** \$485.57 **l** \$557.67
m \$349.06 **n** \$703.12 **o** \$3171.66

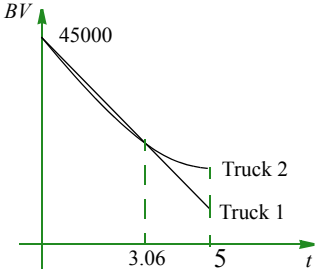
Exercise 22.5 Miscellaneous questions

1 \$1305
2 \$1082
3 \$67
4 9
5 \$14815.44
6 \$65.93
7 \$17442
8 \$3473
9 14.9%
10 \$20600
11 **a** \$4000 **b** **i** 9.41% **ii** 8.02%
12 8
13 \$3
14 \$15000
15 5
16 \$24015
17 £102907.72
18 \$591
19 £8477
20 \$17248
21 **a** **i** £1600 **ii** £5600 **b** 5 years 11 months 3 days **c** 9.17% **d** \$125
22 **a** 13949 Elysian francs **b** \$245.90 **c** 163.69 Florian marks **d** £5.47 Martian
23 **a** 343200 Elysian francs **b** 26400 Elysian francs **c** 3484800 Elysian francs
24 **a** 893.71 ~ 894 francs **b** 208.92 ~ 209 francs **c** 383.02 ~ 383 francs
25 8433 Elysian francs; AUD\$2345; \$55
26 \$1800
27 **a** 4.16% **b** 3.489 months (3 months 15 days)
28 compound interest = \$764.89, simple interest = \$528
29 \$5043.96 ~ \$5044
30 \$5405.26; \$3005.26
31 \$1358.13
32 \$1058.65
33 14.65%

34 Option A: \$3900, Option B: \$4659.51, Option C: \$4644.91. Option B is the best option.
35 \$18381.73
36 3.18 years
37 147.29 fortnights
38 **a** Shortfall of \$1231.08 **b ii** $n = 37.31$, need 38 quarters, i.e. 9.5 years.
39 **a** \$70000 **b** $t = 0$, BV = \$350000; $t = 1$, BV = \$280000; $t = 2$, BV = \$210000; $t = 3$, BV = \$140000; $t = 4$, BV = \$70000; $t = 5$, BV = \$0 **c** $\text{BV} = 350000 - 70000t,\ 0 \le t \le 5$ **d** scrap value = \$0
40 \$3450
41 \$950
42 **a** \$11549 **b** \$5480
43 **a**

	0	1	2	3	4	5
Truck 1	45000	36900	28800	20700	12600	4500
Truck 2	45000	34650	26680.5	20544	15819	12180

b

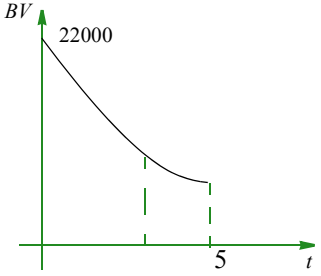


c $0 \le t < 3.06,\ \text{BV}_1 > \text{BV}_2$

$3.06 < t \le 5,\ \text{BV}_2 > \text{BV}_1$

$t = 3.06,\ \text{BV}_1 = \text{BV}_2$

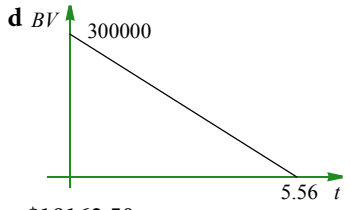
44 **a** Use the equation $\text{BV} = 22000 \times (0.82)^t,\ 0 \le t \le 5$ to generate the scchedule.
b



c $\text{BV} = 22000 \times (0.82)^t,\ 0 \le t \le 5$
d i \$14792.80 **ii** \$10452.61
e \$8156.28

45 **a** \$0.295 per km **b** \$17300 **c** Use $\text{BV} = 35000 - 0.295N,\ 0 \le N \le 100000$ to generate the schedule.
46 **a i** \$138000 **ii** \$270000 **b** Use the equation $\text{BV} = 300000 - 54000t$ to generate the schedule. **c** $a = 300000$, $b = 54000$

d



e 5.56 years

47 **c** \$18162.50
48 **a** \$1042.01; \$42.01 **b** \$122.22 **c** \$736.01

22.6 Graded revision questions

LEVEL 1

1 **a** US\$25 = 3808.75 Elysian francs **b** 50 Florian marks = \$165.51 Utopian **c** 150 Elysian francs = \$1.89 Utopian
2 \$260.75

LEVEL 2

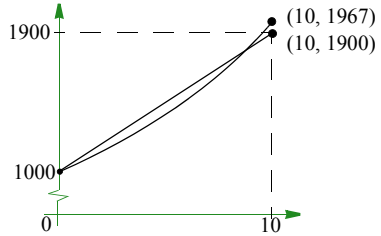
1 \$26800
2 \$1049.91

LEVEL 3

1 Interest = £701.30; equivalent rate, $R = 7.20\%$ p.a
2 **a** 1.0% **b i** \$3000 **ii** \$250 **c** 36 months (or 3 years)
3 **b** 7.113 ~ 7 years and 5 weeks

LEVEL 4

1



For $0 \le t \le 8.07$ simple interest has a higher return.
After 8.07 years, compound interest does better.

2 **Bond A**: 16% p.a. compounded monthly. **Bond B**: 16.2% p.a. compounded semi-annually.
Bond A: effective rate is 17.23%. Bond B: effective rate is 16.86%.
Bond A is the better investment.

22.7 Topic test

1 **a** US\$35 **b** \$128.57 Utopian
2 **a** 960 **b** 1167.55 crowns
3 5527.2 Elysian francs; \$1455.33 Utopian; commission is 1500 – 1455 = \$45
4 1.946 ~1.95 years
5 Option A: i.e. 8.299% ; Option B: i.e. 8.264%. Therefore, Option A is the better investment.
6 **a** 10000 **b** 5 years **c** \$8000 **d** \$1600 per year **e** $BV = 10000 - 1600t,\ 0 \le t \le 5$ **f** \$4666.67
7 **a i** \$11865.23 **ii** \$5005.65 **b i** $a = 50000$, $b = 0.75$ **ii** \$3754.23

Exercise 23.1

1 **a** $\frac{3}{4}$ **b** $\frac{3a}{4b}$ **c** -1 **d** 1 **e** $-\frac{15}{8}$ **f** 0

2 **a** 4 **b** 0.2 **c** 0.027 **d** 0.433 **e** -0.01 **f** -30 **g** 6.2 **h** 1

3 **a** 6 m/s **b** 30 m/s **c** $11 + 6h + h^2$ m/s

4 12 m/s

5 $8 + 2h$

6 -3.49°C/sec

7 **a** 127π cm^3/cm **b i** 19.6667π cm^3/cm **ii** 1.9967π cm^3/cm **iii** 0.2000π cm^3/cm

8 1.115

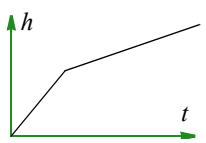
9 **a** -7.5 °C/min **b** $t = 2$ to $t = 6$

10 **a** 8 m **b** 14 m/s **c** average speed **d** 49 m **e** 49 m/s

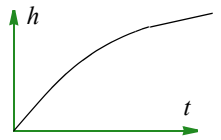
11 **a** \$1160, \$1345.6, \$1560.90, \$1810.64, \$2100.34 **b** \$220.07 per year

Exercise 23.2

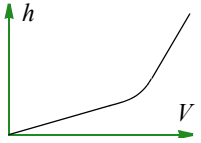
1 **a**



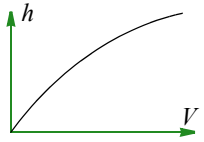
b



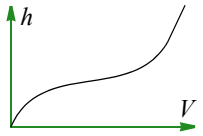
2 **a**



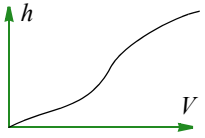
b



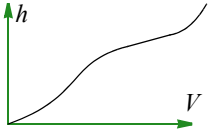
c



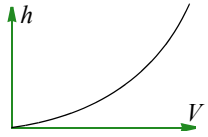
d



e



f



Exercise 23.3

1 **a** $h + 2$ b $4 + h$ **c** $\frac{-1}{1+h}$ **d** $3 - 3h + h^2$

2 **a** 2 **b** 4 **c** -1 **d** 3

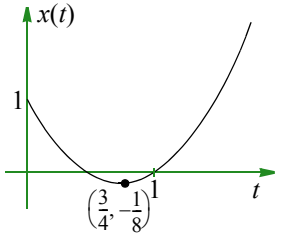
3 **a** $2a + h$ **b** $-(2a + h)$ **c** $(2a + 2) + h$ **d** $3a^2 + 1 + 3ah + h^2$

e $-(3a^2 + 3ah + h^2)$ **f** $3a^2 - 2a + (3a - 1)h + h^2$ **g** $\frac{-2}{a(a+h)}$

h $-\frac{1}{(a-1)(a-1+h)}$ **i** $\frac{1}{\sqrt{a+h}+\sqrt{a}}$

4 **a** 1 ; 1 **b** $2a + h$; $2a$ **c** $3a^2 + 3ah + h^2$; $3a^2$ **d** $4a^3 + 6a^2h + 4ah^2 + h^3$; $4a^3$

5 **a**

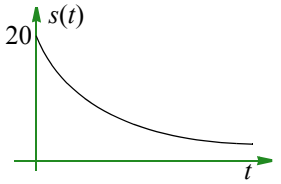


b i 3 ms^{-1} **ii** 2 ms^{-1} **iii** 1.2 ms^{-1}

d Find (limit) as $h \to 0$

e $4t - 3$

6 **a**



b i 20 cm^2 **ii** 17.41 cm^2 **iii** 2.59 cm^2

iv -1.29 cm^2 per day

c $20(1 - 2^{-0.1h})$ cm^2 per day

d i -1.3863 cm^2 per day

ii -1.2935 cm^2 per day

Exercise 23.4

1 **a** 3 **b** 8 **c** $-\frac{1}{9}$ **d** 1.39 **e** -1 **f** $\frac{17}{16}$

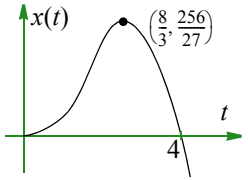
2 4.9 m **b** $4.9(h^2 + 2h)$ m **c** 9.8 m/s

3 **a** $8x$ **b** $10x$ **c** $12x^2$ **d** $15x^2$ **e** $16x^3$ **f** $20x^3$

4 **a** $4x$ **b** -1 **c** $-1 + 3x^2$

5 **a** 1 ms^{-1} **b** $(2 - a)$ ms^{-1}

6 **a**



b i 5 ms^{-1} **ii** 4 ms^{-1} **c** $8t - 3t^2$ ms^{-1} **d** $\frac{8}{3}$ sec

Exercise 23.5 Miscellaneous questions

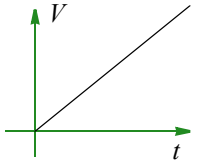
1 **a** $\frac{1}{2}(\sqrt{3} - 1)$ **b** 0

2 13.5 bact. per hour

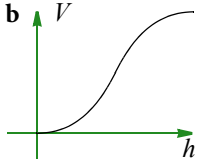
3 **a** 122.5 m **b** 24.5 ms^{-1}

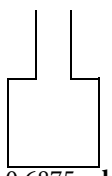
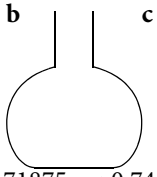
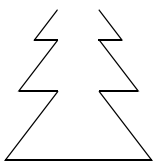
4 **a** $\frac{2}{9}$ mg/mL/hr **b** $\frac{1}{8}$ mg/mL/hr **c** $-\frac{5}{72}$ mg/mL/hr

5 **a**



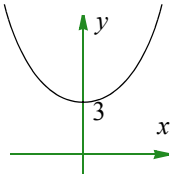
b



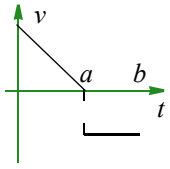
6 **a**  **b**  **c** 

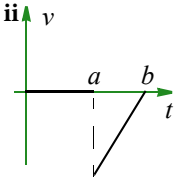
7 **a** 0.6875 **b** 0.71875 **c** 0.74375 **d** $v = 0.75 - 0.0625h$; as $h \to 0$, $v \to 0.75$

8 **a** –1 **b** –1

9 **a** 

b i $2xh + h^2$ **ii** $2x + h$ **c** $2x$

10 **a i** 

b ii 

c iii $\leftrightarrow \alpha$

11 **a i** –15 **ii** –29 **iii** –14 **b i** $-15 - 12h - 2h^2$ **ii** $-12h - 2h^2$ **iii** –14 **iv** –12

12 **a** Need **accurate** plot **b i** –7 **ii** 0 **iii** This answer will depend on the accuracy of your plot. However, you will not be able to obtain an algebraic expression for this answer (unless you can deduce the equation of the function). Again, part **d** will depend on the accuracy of you plot.
e gradient of tangent = gradient of curve at the same point.

13 **a** 40 cm^3 **b** –0.7692 **c i** 140 cm^3 **ii** 0 cm^3 **d** $t = 7.5$
e i Answers will vary, depending on the accuracy of your plot. **ii** 0

23.6 Graded revision questions

LEVEL 1

1 **a** 2 **b** –1 **c** –0.5

2 **a** 0.5 **b** –0.6 **c** 0

3 **a** A, D **b** B, C **c** C, D **d** A, B

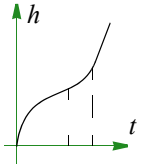
LEVEL 2

1 **a** –4 **b** –2

2 0.2, measures average rate of change of $f(x)$ over the interval [3, 8].

3 **a** –6 °C/min **b** –0.0193 °C/min

4 **a** $2xh + h^2$ **b** $2x + h$

5 

LEVEL 3

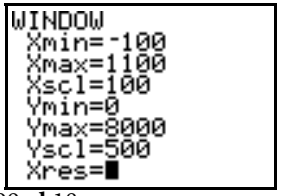
1 **a** $-\frac{4}{1+2h}$ **b** $1 + h$

2 **a** –4 **b** 1

3 $-3a^2 - 3ah$; $-3a^2$

4 **a** $3x^2$ **b** $-\frac{1}{x^2}$

LEVEL 4

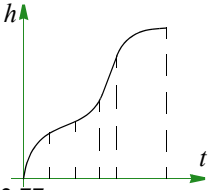
1 **a i** $0 < x < 1000$ **ii** 


b i 4500 **ii** 0 **c** 599 **d** 18

2 **a i** $10h - 4ht - 2h^2$ **ii** 2 **b** –4

3 1.829

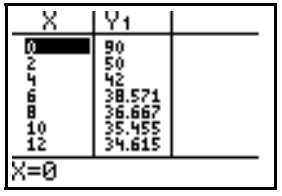
23.7 Topic test

1 

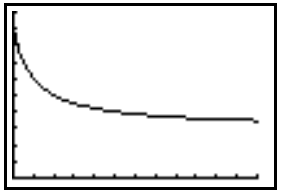
2 2.77

3 0

4 **a** Use graph paper to plot and sketch **b** –1.03 **c** Last decade
d ii 7.4 deaths per 100 000 **e i** $2aht - bh + ah^2$ **ii** –0.64

5 **a** 

b –20°C/min **c i** $-\frac{20}{h+3}$ **ii** $-\frac{20}{3}$ °C/min

d i  **ii** $T \to 30$

Exercise 24.1

1 **a** $5x^4$ **b** $9x^8$ **c** $25x^{24}$ **d** $27x^2$ **e** $-28x^6$ **f** $2x^7$ **g** $2x$ **h** $20x^3 + 2$
i $-15x^4 + 18x^2 - 1$ **j** $-\frac{4}{3}x^3 + 10$ **k** $9x^2 - 12x$ **l** $3 + \frac{2}{5}x + 4x^3$

2 **a** $2x + 2$ **b** $4x^3 + 3x^2 - 1$ **c** $3x^2 + 1$ **d** $\frac{1}{x^2}$ **e** $4x^3 - 12x^2 + 8x$ **f** $\frac{1}{2} + x$

3 **a i** $2(3x-1)$ **ii** $9(3x-1)^2$ **b i** 3 **ii** $18x - 6$ **iii** $9(3x-1)^2$ **c** $3n(3x-1)^{n-1}$

4 $a = 1, b = 1$

5 **a i** $2x - \frac{8}{x^3}$ **ii** $2 + \frac{24}{x^4}$ **b i** $1 + \frac{1}{x^2} - \frac{4}{x^3}$ **ii** $-\frac{2}{x^3} + \frac{12}{x^4}$ **c i** $\frac{6}{x^2} - \frac{18}{x^3}$ **ii** $-\frac{12}{x^3} + \frac{54}{x^4}$
d i $1 + \frac{8}{x^5}$ **ii** $-\frac{40}{x^6}$ **e i** $-\frac{4}{x^3} + \frac{3}{x^2} - \frac{15}{x^4}$ **ii** $\frac{12}{x^4} - \frac{6}{x^3} + \frac{60}{x^5}$ **f i** $3x^2 - \frac{6}{x^3} + \frac{30}{x^6}$
ii $6x + \frac{18}{x^4} - \frac{180}{x^7}$

6 $a = 1, b = 1$

7 **a i** $2x - \frac{8}{x^3}$ **ii** $2 + \frac{24}{x^4}$ **b i** $1 + \frac{1}{x^2} - \frac{4}{x^3}$ **ii** $-\frac{2}{x^3} + \frac{12}{x^4}$ **c i** $\frac{6}{x^2} - \frac{18}{x^3}$ **ii** $-\frac{12}{x^3} + \frac{54}{x^4}$
d i $1 + \frac{8}{x^5}$ **ii** $-\frac{40}{x^6}$ **e i** $-\frac{4}{x^3} + \frac{3}{x^2} - \frac{15}{x^4}$ **ii** $\frac{12}{x^4} - \frac{6}{x^3} + \frac{60}{x^5}$ **f i** $3x^2 - \frac{6}{x^3} + \frac{30}{x^6}$ **ii** $6x + \frac{18}{x^4} - \frac{180}{x^7}$

Exercise 24.2.1

1 $m_{PQ} = 4 + h$; $\lim_{h \to 0} m_{PQ} = 4$

2 **a** $P(1, 1), Q\left(1 + h, \frac{2}{2+h}\right)$; $m_{PQ} = -\frac{1}{2+h}$; $\lim_{h \to 0} m_{PQ} = -\frac{1}{2}$

3 **a** -12

4 **a** 3 **b** $-\frac{1}{4}$ **c** 12 **d** 4

5 **a** $\pm\sqrt{\frac{8}{3}}$

6 **a** $2x - 12$ **b** -18 **c** $(8, -32)$

7 **a** $-3x^2 + 3$ **b** 0 **c** $(\pm\sqrt{2}, \pm\sqrt{2})$

8 **a** $\left(\pm\frac{\sqrt{2}}{2}, -\frac{1}{16}\right), (0,0)$ **b** $\left\{x : \frac{-1}{\sqrt{2}} < x < 0\right\} \cup \left\{x : x > \frac{1}{\sqrt{2}}\right\}$

9 **a** $x = \frac{1}{3}, -1$

10 **a** $-2, 6, 3$ **b** -2

11 **a** $a = -3$ $b = 1.5$

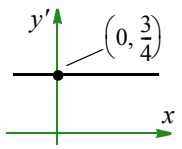
12 **a** $f'(a+b) = 2(a+b) = 2a + 2b$

13 **a** $4a^2 - 2a^4$ **b** $4 - 4a^2$

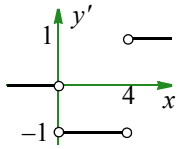
14 **a** -56

Exercise 24.2.2

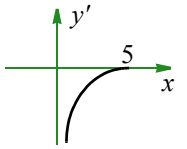
1 **a**



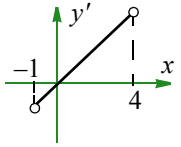
b



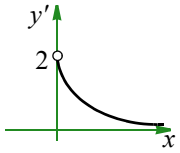
c



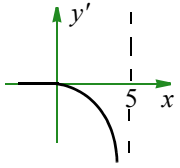
d



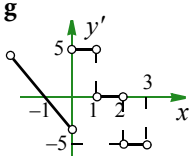
e



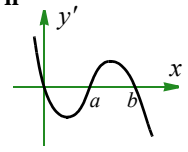
f



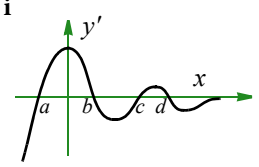
g



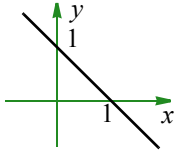
h



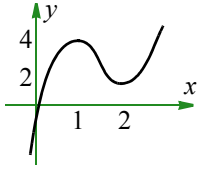
i



2



3



Exercise 24.3

1 **a** $y = 7x - 10$ **b** $y = -4x + 4$ **c** $y = 6x - 16$ **d** $y = 2x - 9$ **e** $y = -6x + 2$
f $y = -3.5x + 7$ **g** $y = 28x - 48$ **h** $y = 8x - 12$

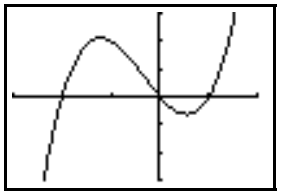
2 A: $y = 28x - 44$, B: $y = -28x - 44$, Isosceles. $z \equiv (0, a^2 - 3a^4)$

3 $y = 4x - 9$

4 $y = -13, y = 19$

5 **a** A(−3, 7), B(4, 0) **b** A: $y = -8x + 32$, B: $y = 6x + 25$, $\left(\frac{1}{2}, 28\right)$

6 **c** $m = -2, n = 5$

7 **a**  **b i** $y = -2x$ **ii** $y = 6(x+2)$ **c** 3 sq. units

8 **a** $a = 1, b = 2, c = 4$ **b** $(0.5, -2)$ **c** $\sqrt{11.25} + \sqrt{38.25} + \sqrt{18} \approx 13.7814$

9 $3y = 21x + 31$; $3y = 21x - 77$

10 **a** $y = 3$ **b** $y = 2x$

11 $a = -1$

12 a $7y + x = 30$ b $4y + x = 1$ c $x = -1$ d $x = 1$ e $3y + x = 11$
f $48y + x = 386$ g $48y - x = -194$ h $(m+n)y + x = 1 + mn(m+n)$

13 Tangent: $y = 2$; Normal: $x = 1$; Area = 2 sq units

14 a At (1, 0), $3y - x = -1$. At (4, 0), $3y + x = 4$ (b) $5y - x = 20$

15 a $8y + x = 0$; $4y - x = -2$; $8y + x = 4$ (b) $8y + x = 0$

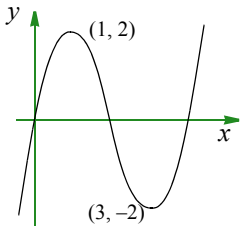
16 $Q \equiv \left(-\frac{31}{6}, \frac{253}{36}\right)$

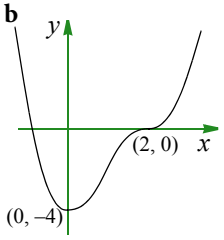
17 a i $6y - x = 53$ ii $3y + x = 29$ iii $6y + x = 53$ iv $3y - x = 29$

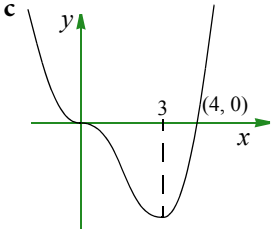
b i $R \equiv \left(0, \frac{53}{6}\right)$ ii $S \equiv \left(0, \frac{29}{3}\right)$ c $\frac{25}{18}$ sq units

18 $C \equiv \left(\frac{1}{3}, \frac{13}{3}\right)$

Exercise 24.4

1 **a** 

b 

c 

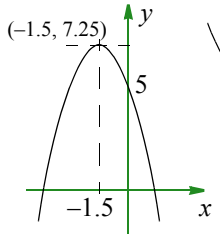
2 **a** max at (1, 4) **b** min at $\left(-\frac{9}{2}, -\frac{81}{4}\right)$ **c** min at (3, –45) max (–3, 63)

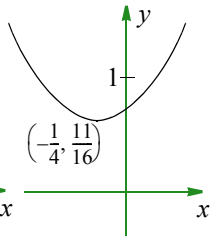
d max at (0, 8), min at (4, –24) **e** max at (1, 8), min at (–3, –24)

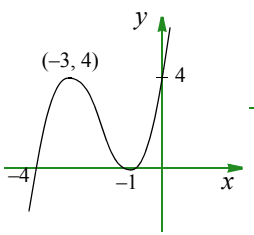
f min at $\left(\frac{1+\sqrt{13}}{3}, \frac{70-26\sqrt{13}}{27}\right)$, max at $\left(\frac{1-\sqrt{13}}{3}, \frac{70+26\sqrt{13}}{27}\right)$

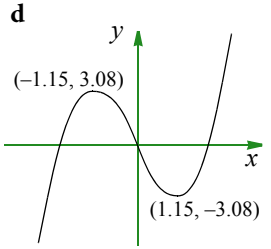
g min at (1, 0) max at $\left(-\frac{1}{3}, \frac{32}{27}\right)$ **h** max at (0, 16), min at (2, 0), min at (–2, 0)

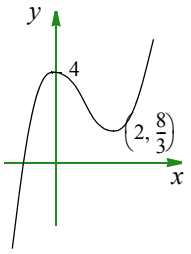
3

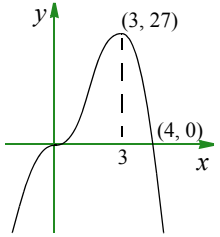
a 

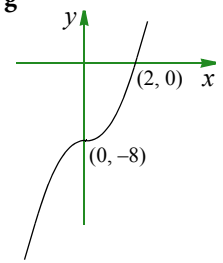
b 

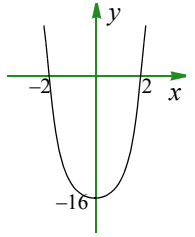
c 

d 

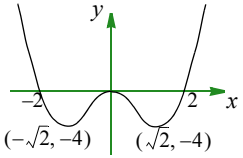
e 

f 

g 

h 

4 min. at (1, –3), max. at (–3, 29), non-stationary infl. (–1, 13)

5 

6 **a** 8 **b** 0 **c** 4 **d** $27\sqrt[3]{9} \approx 56.16$

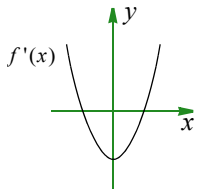
7 **a** min value –82. **b** max value 26

8 **a** A **i** Yes **ii** non-stationary pt. of inflect.
B. **i** Yes **ii** Stationary point (local/global min.)
C. **i** Yes **ii** non-stationary pt. of inflect.
b A. **i** No **ii** Local/global max.
B. **i** No. **ii** Local/global min.
C. **i** Yes **ii** Stationary point (local max.)
c A. **i** Yes **ii** Stationary point (local/global max.)
B. **i** No **ii** Local min.
C. **i** Yes **ii** non-stationary pt. of inflect.
d A. **i** Yes **ii** Stationary pt. (local/global max.)
B. **i** No **ii** Local min.
C. **i** Yes **ii** Stationary point (local max.)
e A. **i** No **ii** Cusp (local min.)
B. **i** Yes **ii** Stationary pt. of inflect.

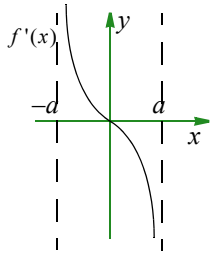
C. **i** No **ii** Local max.
f A. **i** Yes **ii** Stationary point (local/global max.)
B. **i** Yes **ii** Stationary point (local/global min.)
C. **i** No **ii** Tangent parallel to y-axis.

9 **a i** A **ii** B **iii** C **b i** C **ii** B **iii** A

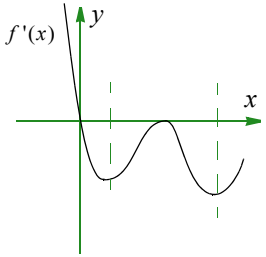
10 **a**



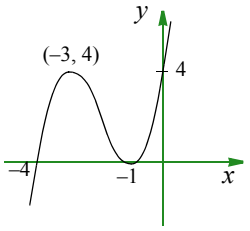
b



c



11



$y = x^3 + 6x^2 + 9x + 4$

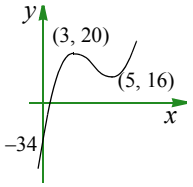
12 $f(x) = \frac{1}{3}x^3 - x^2 - 3x - 6$

13 $f(x) = 3x^5 - 20x^3$

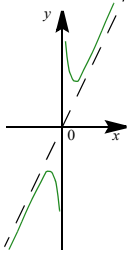
14 $m = -0.5$, $n = 1.5$

15 $a = 2$, $b = -3$, $c = 0$

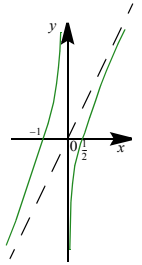
16 $a = 1$, $b = -12$, $c = 45$, $d = -34$



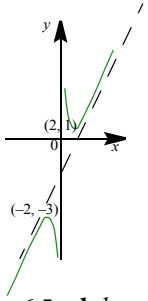
17 **a i** $x = 0$ **ii** (2, 4) and (−2, −4)



18



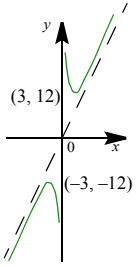
19



20 **a** 6.5 **b** $k > -8.5$

21 (2, 12)

22 **a i** (3, 12); (−3, −12) **ii**

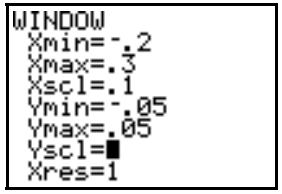


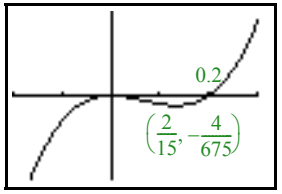
b $x = 3$, $y = 12$

Exercise 24.5

1 (−2, 21)

2



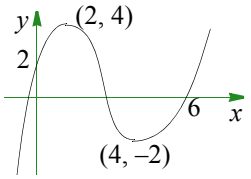


3 (3, 9); $\left(-1, \frac{5}{3}\right)$

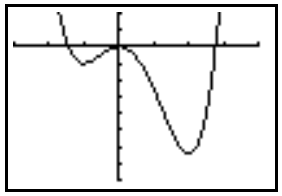
4 $k = -2, 4$

5 $y = 6x + 5$; $y = 6x - 27$

6

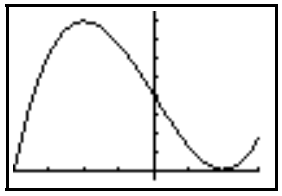


7 **a** (0, 0); (–1, –5); (2, –32) **b**



8 **a** $-\frac{1}{3}$ **b** $-\frac{1}{3} < x < 1$ **c** $x < -\frac{1}{3}$ or $x > 1$

9 **a**

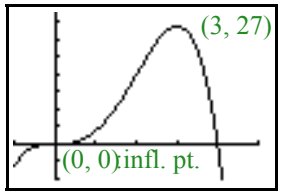


b i 32 **ii** 0 **c i** $k = 32$ **ii** $7 < k < 32$, $k = 0$ **iii** $0 < k \leq 7$

10 **a** 0, 2 **b i** $y = -9x + 27$ **ii** $y = 4$ **c** $\left(\frac{23}{9}, 4\right)$

11 **a** $x < -\frac{5}{3}$ or $x > 1$ **b** $-\frac{5}{3} < x < 1$ **c** $x = -\frac{5}{3}$

12 **a**



b $y = 16x - 16$

13 **a i** $\frac{dy}{dx} = 3x^2 + 6x + 3$ **ii** 48 **b i** no **ii** $x < -1$ or $x > -1$

c no local max. or local min., only one stationary point of inflection at (–1, 5)

14 $a = 4$, $b = 2$

15 **a** $y = -12x + 12$ **b** $y = 16x - 15$

16 $a = 1$, $b = -8$

Exercise 24.6 Miscellaneous questions

1 22.6 m

2 **a** 1.5 kmh^{-1} **b** \$19.55 per km

3 **a** 400 **b** \$46 400 000

4 \$273.86

5 0.45 m^3

6 5 m by 5 m

7 128

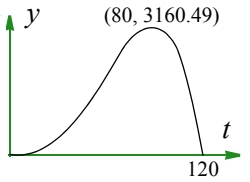
8 **a** 10.5 **b** 5.25

9 72

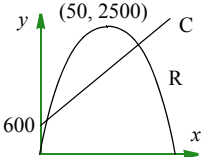
10 **a** $y = 100 - 2x$ **b** $A = x(100 - 2x)$, $0 < x < 50$ **c** $x = 25$, $y = 50$

11 **a** $\frac{100}{x} - \frac{1}{2}x$, $0 < x < 10\sqrt{2}$ **b** $\frac{2000}{9}\sqrt{6} \approx 544.3$ cm^3

12 **a** 400 mLs^{-1} **b** 40 sec **c**



13 **a**



b 8.38, 71.62 **c** $9 \leq x \leq 71$ **d** $80x - x^2 - 600$, \$1000

14 4 by $\frac{8}{3}$

15 $348 - 8\sqrt{170} \sim 243.7$ cm^2

16 2

17 radius = $\sqrt{\frac{10}{3}}$ cm, height = $2\sqrt{\frac{10}{3}}$ cm

18 $3\sqrt{\frac{15}{\pi}}$

19 5 cm

20 **a** $h = \frac{24r^2}{r^2 - 144}$ **b** $\frac{8\pi r^4}{r^2 - 144}$ **c** $r = 12\sqrt{2}$, $h = 48$

21 2 : 1

22 $\frac{10}{\sqrt{3\pi}}$

23 **a i** $\pi r^2 h + \frac{2}{3}\pi r^3$ **ii** $3\pi r^2 + 2\pi rh$ **c** $r : h = 1:1$

24.7 Graded revision questions

LEVEL 1

1 **a** $7x^6$ **b** $12x$ **c** -1

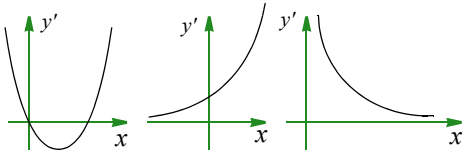
2 **a** $2x$ **b** $12x - 1$ **c** $2x$

3 **a** $3x^2 - 2x$ **b** $-\frac{2}{x^2} - 1$ **c** $4x^3 - 12x$ **d** $1 - \frac{1}{x^2}$ **e** $81 - 36x^2 + 3x^2$ **f** $3x^2 + \frac{3}{x^4}$

LEVEL 2

1 **a** 3 **b** –27

2 $(0, 0)\ (\pm\sqrt{2}, -8)$
3 $y = x$
4 2
5



6 $x = 1,\ y' = -1$; $x = 2,\ y' = 1$
7 $4y + x = 15$

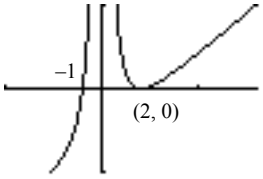
LEVEL 3

1 **a** $3x^2 - 3$ **b** -3 **c** $(\pm 1, 0)$
2 $y = -2x + 6$; $y = 2x + 6$
3 $\frac{6}{x^3}$
4 $a = -2,\ b = 4$
5 0, 2
6 **a** $y = -3x + 4$ **b** $y = -3x - 12$

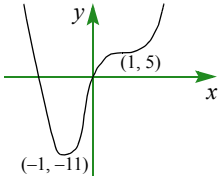
LEVEL 4

1 **a** $f'(x) = 1 - \frac{8}{x^3}$ **b i** $-1 \le x < 0$ or $x > 0$ **ii** $x < 0$ or $x \ge 2$

iii



2 **a** $x^3 - x^2 - ax + a$ **b** $y' = 12(x^3 - x^2 - ax + a)$ **c** $x = \pm\sqrt{a}$ or $x = 1$
d i Local min. at $x = -\sqrt{a}$ and $x = 1$; local min. at $x = \sqrt{a}$
ii Stationary point of infl. $x = 1$; local min. at $x = -1$
e $y = 3x^4 - 4x^3 - 6x^2 + 12x$

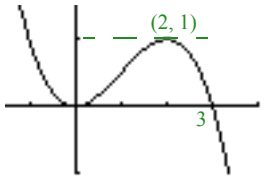


3 **a** (1, 3) **b i** $y = 3.5x - 2$ **ii** (−0.5, −3.75)

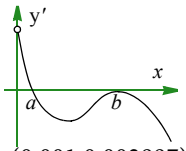
24.8 Topic test

1 **a i** 9 **ii** $9 + 8h + 2h^2$ **b** $8 + 2h$ **c** 8
2 **a** $-6x^2 + 24$ **b** $-6a^2 + 24$ **c** (1, −10) and (−1, −54)
3 $b = 1$

4 **a**



b $4k$
5 $6x - 8$
6 $y = 6x - 8$
a i global max. **ii** $0 < x < a$ **iii**



b i (0.1, 0.271); (0.01, 0.0297); (0.001,0.002997) **ii** −3

Revision Set A

1 **a** No **b** Yes **c** Yes
2 **a** 6.25×10^{-4} **b** 1.0×10^{-2} **c** 8.1×10^{-7}
3 **a** 1.39×10^{0} **b** 1.96×10^{2} **c** 2.48×10^{-2} **d** -6.27×10^{7} **e** 9.85×10^{-1}
4 13.9681% ~ 14%
5 **c** 21°C hr^{-1} d $O(20)_{min} = 853$, $O(20)_{max} = 1117$;
$O(30)_{min} = 997$, $O(30)_{max} = 1393$; 6.6%
6 [19.25, 29.25]
7 **a** 5% b ~ 14.7% c 10%
8 ~ 0.64%
9 **a** Intercepts:(0, 0) and (4, 0), turning point at (2, 4) **b** I = [2, ∞[c ii]−∞, 4]
10 **b**]0, 4] **c** $x = 2$
11 **b** $k = -1$
12 **a** $f(g(x)) = 9 - x$ **b i** [0, ∞[**iii**]−∞, 9]
13 **a** Yes **c i** $\left[0, \frac{16}{3}\right]$ **ii** [0, 4] **d** $\frac{8}{3}$
14 **a** 8 **b** 6 **c** 6
15 **a i** $\frac{4}{5}$ or $-\frac{2}{3}$ **ii** 0 or $\frac{5}{9}$ **b** −1.22 or 0.55
16 **a i** (4, −2) **ii** 4 **iii** min values = −2 **b** 3 or 5
17 5 cm by 20 cm by 30 cm
18 **a** A: 31.2 ms^{-1}; B: 44.5 ms^{-1} **b** A: 4.6 sec; B: 17.1 sec **c i** 8.325 sec **ii** 40.51 ms^{-1}
19 **a i** \$6000 **ii** 20% **iii** \$26,000 **iv** $m = 0.3$, $c = -3000$ **b i** \$6000
ii $m_1 = 0.4, c_1 = -7750$; $m_2 = 0.2, c_2 = 0$ **c** $30000 < x < 47500$
20 **a** 4 m **b** $h(t) = -5(t - 1.2)^2 + 7.2$, max. height = 7.2 m when $t = 1.2$
c 0.537 sec and 1.86 sec **d** 2.4 sec
21 **a** $5x - \frac{1}{2}x^2$ cm^2 **b** 1.25
22 **a** $a = 12,\ b = 27$ **b** −9

23 **a** $4(s+10)$ km **b** 87.14 km hr^{-1}
24 20 units
25 **a** $50x + 10y > 450$ **b** $y = 26 - x$ **c** 5
26 **a** $a = 0, b = 10$ **b** 50 m^2
27 **a** 0.5 **b** $F = 0.00125x, 0 \le x \le 80$ **c** $y = 0.000625x(x-80)^2, 0 \le x \le 80$ **e** 26.67 m

Revision Set B

1 **a** 5 **b** 390
2 **a** 189 **b** 99 **c** −96
3 **a** $A = 10$ **b** $k = 0.009691$
4 1
5 **a i** 7m **ii** 4m **iii** 1m **c** 2.35, 9.65, 14.35, 21.65
d 9:39 am to 2:21 pm and 9:39 pm to midnight **e** $h(t) = 3\cos\left(\left(\frac{360}{13}t°\right) + 4\right)$
6 **a i** \$17680 **ii** \$130 000 **b i** \$19618.12 **ii** \$1325999.37 **c** in the 5th year
7 **b i** 28 **ii** 40 **iii** 10
8 **a** 0.12 **b** $\frac{14}{23}$
9 **a** 0.89 **b** 0.525 **c** 0.4494
10 **a** 0.46 **b i** 0.60 **ii** $\frac{9}{23}$
11 **a** 341.99 (approx.) **b** 20 terms
12 **a ii** 2000 **c** 52 hours **d** 176995
13 **a** 23 **b** 3 **c** 7 **d** 26
14 **b ii** −2.51
15 20 rows
16 **a** P = {3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39}; R = {1, 2, 3, 4, 6, 9, 12, 18, 36}; Q = { 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38} **b i (a)** x is not a multiple of 3 between 0 and 40
b x is a multiple of 3 and an even number between 0 and 40
c x is a multiple of 3 and an even number and a factor of 36 between 0 and 40
d x is either a multiple of 3 between 0 and 40 or an even number between 0 and 40 or a factor of 36
e x is not a multiple of 3 between 0 and 40 and x is not an even number between 0 and 40
f x is not a multiple of 3 between 0 and 40 and x is not an even number between 0 and 40 and x is not a factor of 36
g x is not an even factor of 36 that is a multiple of 3
h x is an even number or a factor of 36 but is not a multiple of 3.
iv a {1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20, 22, 23, 25, 26, 28, 29, 31, 32, 34, 35, 37, 38}
b {6, 12, 18, 24, 30, 36} **c** {6, 12, 18, 36}
d {2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 26, 27, 28, 30, 32, 33, 34, 36, 38, 39}
e {1, 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, 35, 37}
f {5, 7, 11, 13, 17, 19, 23, 25, 29, 31, 35, 37} **g** U\{6, 12, 18, 36}
h {1, 2, 4, 8, 10, 14,16, 20, 22, 26, 28, 32, 34, 38}
17 **a** $\frac{1}{4}$ **b** $\frac{3}{8}$ **c** 0.3169
18 $\frac{64}{425}$
19 $\frac{10}{21}$
20 **b i** $k = 0, 6$ **ii** $k = 3$
21 **a** $(4-\pi)$ cm^2 **b** $\frac{1}{2}(4-\pi)$ cm^2 **c ii** 0.5 **iii** $A_n = (4-\pi)\times\left(\frac{1}{2}\right)^{n-1}, n = 1, 2, \ldots$
iv $\frac{1}{16}(4-\pi)$ cm^2 **d i** $\frac{31}{16}(4-\pi)$ cm^2 **ii** $2(4-\pi)$ cm^2 **e** Geometric sequence
22 **a–c** Neither **d** contradiction **e–f** tautology **g** contradiction **h** neither
23 **a** 0.02 **b** 0.06 **c** 0.225 **d** 0.635
24 **b** 6 **c** 18
25 **a i** $\frac{7}{3}, \frac{11}{3}$ **ii** $a = \frac{7}{3}, b = \frac{11}{3}$ **b i** $k = -2$ **ii** $-2 < k < 2$
26 **a** 0.8 **b** 0.25
27 **a** $\frac{4}{15}$ m **b** 48 m
28 **b** $a = 1, b = 9, c = -4.5, d = -4$
29 **a i** 1 a.m. and 7 a.m. **ii** 0.9 m **b** 15 hours **c i** $a = 1.5, b = 45, c = 2$ **ii** 2 m
d i 6 times **ii** 43.75%
30 **a** $\frac{2}{3}$ **b** $\frac{2}{9}$ **c** Not independent
31 **a** \$50000 **b** \$5801 **c** 2 years 7 weeks (approx.)
d between 2.8 years and 3.7 years **b** $k = 8839.71$

Revision Set C

1 **a** $\sqrt{61}$ **b** $y = 3x$
2 **a i** (15, 0) **ii** (0, 8) **b** 17
3 **a i** {3, 8} **ii** {4, 6} **b** 2
4 13 km S22°37′ W
5 **a i** 7.2 cm **ii** 8.8 cm **b i** 56.3° **ii** 62.9°
6 **a i** $\left(\frac{1}{2}, \frac{1}{2}\right)$ **ii** $\sqrt{34}$ **b i** $3x - 5y + 1 = 0$ **ii** $5x + 3y = 0$
7 **a** 120° **b** 24.25 cm^2
8 **a i** 0.5196 m **ii** 0.3464 m **b** 1.15 m **c** 73°13′
9 **a** −3 or 5 **b** 0.5 **c** (0, 4)
10 21.0 cm
11 **a** 1 **b** (2, 2) **c** $\sqrt{40}$ **d** $y = x - 4$ **e** (3, 1)
12 **a** 8 cm **b** 28°4′
13 **b i** 660 m **ii** 688 m
14 906 m

15 **a** $38°40'$ **b** 0.08004 cm^2 **c** 6.05 cm^2 **d** \$493.71
16 **a** A(0, 6); B(3, 4); C(−3, 4); D(−3, 3); E(3,3); F(−1, 1); G(1, 1) **b i** $y = x$
ii $y = -x$ **c** $y = -\frac{2}{3}x + 6$ **d** 5 **e i** 135° **ii** $112°38'$
17 1.262 ha
18 1623 m
19 1939 m
20 **a** 1171 m **b** 1.54 km
21 **a** 7.2 cm **b** $67°58'$
22 **b** 2 **c** $y = 2x - 3$ **d** $2x + 4y = 3$
23 **a** 16.97 cm **b** 13.56 cm **c i** 66.14° **ii** 66.06°
24 $60°36'$
25 **a** $4\sqrt{13}$ **b** 1.5 **c** $56°19'$ **d** $3x - 2y + 3 = 0$ **e i** P(−1, 0) **ii** Q(0, 1.5) **f** $y = 1.5x$
26 **b i** $y = -x$ **c** (5, −5)
27 **b** $5\sqrt{10}$
28 **a** 120° **b** 50 m
29 **a** 12 m **b** 35 m **c i** $\frac{7}{22}$ m **ii** 1039.5 m^3
30 **a i** 22.59° **ii** 39.81° **b** 100.1°
31 **c** $x = 2.40$

Revision Set D

1 **a** 70 **b** 30 **c** 90 **d** 55
2 **a i** 20 **ii** 3 **b i** 12 **ii** 12
3 **a** 3489.15 francs **ii** 53.74 marks
4 US\$291
5 20160 lire
6 **b** $r = 0.9843$, yes **c** 96.89% **d i** $y = 5.1037x + 447.2070$ **e** 702.4 kg **f** 30.46 kg
7 **a i** Total charges = \$3.20 **ii** Total balance = \$7777.31 **iii** Equity = \$17267.66
b ii \$2135.48 **iii** 2 years 9.5 months – mid-February 2000
8 **a** $a = 57.19$, $b = 61.76$, $c = 17.81$, $d = 19.24$ **b i** H_0: Student results are independent of teaching styles vs H_1: Student results are dependent on teaching style used
c 2.3786 **d i** 2 **ii** 5.991 **iii** Accept H_0
9 4863.22 Swiss francs
10 2341.50 crowns
11 B
12 **a** mean = \$33.23 k, mode = \$32K, median = \$32k Distribution is positively skewed **b** \$6.74 k **c** workers on 35 k, 50 k, 45 k, 55 k **d** Managers **e** \$0.87 k **f** \$997 per worker **g** 3.009% increase **h** 0.4268
13 **a i** 0.54 **ii** 0.70 **iii** 0.54 **b i** \$1315.79 **ii** \$1289.47 **c i** 17.45 years
14 **a** Reject H_0 **b** same as **a** (but at the 1% level of sig.)
15 **a i** 50.23 **ii** $b = 34.77$, $c = 26.77$, $d = 13.23$, $e = 23$ **b i** H_0: Type of car sold is independent of location of showroom. **ii** H_1: Type of car sold is dependent of location of showroom. **c i** Chi-square test for independence
iii Reject H_0 at the 5% level of significance
16 **ii** 6.35 g; 2.615 g
17 **a** 0.5 **b** $y = 0.25x + 57.5$ **c** E **d** $y = \frac{43}{9}x - 12$
18 **b** [1200, 1400[**c** median = 1340 **d** 1307 hours; 400.79 hours

Revision Set E

1 **a** $-\frac{2+h}{(1+h)^2}$ **b** −2
2 **a** Stationary points: (0, 0), $\left(\pm\frac{1}{\sqrt{2}}, 1\right)$
3 Intercepts: (0, 0) and (4, 0); Stationary points: (0, 0) and (3, 3)
4 **a** (1, −2), (−1, 4), (3, 0)
c At (1, −2) tangent is $y = -x - 1$ and at (3, 0) tangent is $y = 3x - 9$
5 **a** 9 **b** $y = 9x - 9$
6 **a i** $3x^2 + 10x + 7$ **ii** $6x + 10$ **b** $x < -\frac{7}{3}$ or $x > -1$
7 **a** $12 + 6h + h^2, h \neq 0$ **b** 12
8 **a** 5.39 **b i** $\frac{1}{2}\cos\left(\frac{1}{4}t\right)$ **ii** −0.40
9 $a = -1$, $b = 6$ and $c = -9$
10 **a** [0, 5] **c** 0.625 **d** $\frac{1}{10}(5 - 2t)$ ms^{-2}
11 **a** $1 - \frac{1}{t^2}$ **b** $y = 0$
12 −0.2
13 **a** Stationary point of infl. at (0, 0); local min at $\left(-\frac{3}{4}, -\frac{27}{128}\right)$ **b** 0, −1
d $\left\{x: -\frac{3}{4} < x < 0\right\} \cup \{x: x > 0\}$
14 **b** $y = 10 - 4t$, i.e. C **c** Boy at $(20 - 3t, 0)$ and girl at $(0, 10 - 4t)$ **d** $k = 200$ **e** $t = 4$
15 $\frac{11}{9}$
16 **a** 0, $\frac{3 \pm \sqrt{105}}{4}$ **b** local max. at (−1, 14); local min. at (2, −40) **d** $y = -24x - 2$
e $\frac{1}{12}$ sq. units
17 local min. at $x = -1$; local max. at $x = 1$
18 **b i** 1 **ii** $y = x$ **c** (3, 3) **d** 36 sq. units
19 **a** 50 ms^{-1} **b** 100 ms^{-1}
20 $y = -4x + 21$
21 (0.5, 0) and (−0.5, 0)
22 **a** $-2 \pm \sqrt{3}$ **b** 2 or −1 **c** At $x = 2$, $m = 8$; at $x = -1$, $m = 2$ **d** $y = 2x$

23 **a** For $f(x)$: 9.6. For $g(x)$: 14.77 **b i** $d(x) = -x^3 + 6x^2 - 9x + 7, 0 \le x \le 4$
ii 7 where $x = 3$ and $x = 0$ **iii** 3 at $x = 1$ and at $x = 4$ **c i** $y = -9x + 39$ **ii** α is an approximation of where $d(x) = 0$